CHAPTER 6

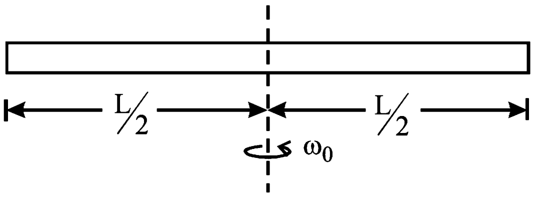
Rotational Motion

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $3a/4$ above the base. The minimum value of F for which the cube begins to tip about the edge is .... (Assume that the cube does not slide). *(1984 - 2 Marks)*

2. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpenducular to the rod and passing through the midpoint of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is .... *(1988 - 2 Marks)*

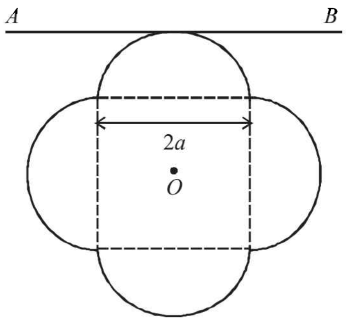


3. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x–y plane) with its axis fixed along the y-axis and free to rotate about its axis. The platform is given a motion in the x-direction given by $x = A \cos(\omega t)$. There is no slipping between the cylinder and platform. The maximum torque acting on the cylinder during its motion is .... *(1988 - 2 Marks)*

4. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$ where A is a constant, r is the instantaneous radius of the circle and n = ... *(1993 - 1 Mark)*

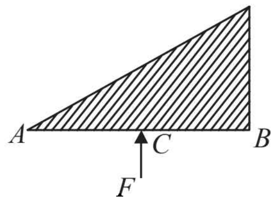
5. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is... and on B is..... *(1997 - 2 Marks)*

6. A symmetric lamina of mass M consists of a square shape with a semicircular section over of the edge of the square as shown in Fig. P-10. The side of the square is $2a$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $1.6\,Ma^2$. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is.... *(1997 - 2 Marks)*



B True / False

1. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through A, (b) passing through B, by the application of the same force, F, at C (midpoint of AB) as shown in the figure. The angular acceleration in both the cases will be the same. *(1985 - 3 Marks)*



2. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$. *(1986 - 3 Marks)*

3. A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and the cylinder. The rolling friction in both cases is negligible. The cylinder will reach the wall first. *(1989 - 2 Marks)*

4. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s. *(1989 - 2 Marks)*

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C MCQs with One Correct Answer

1. A thin circular ring of mass 'M and radius r is rotating about its axis with a constant angular velocity ω, Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity *(1983 - 1 Mark)*

   (a) $\frac{\omega M}{(M+m)}$ (b) $\frac{\omega (M-2m)}{(M+2m)}$

   (c) $\frac{\omega M}{(M+2m)}$ (d) $\frac{\omega (M+2m)}{M}$

2. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of *(1995S)*

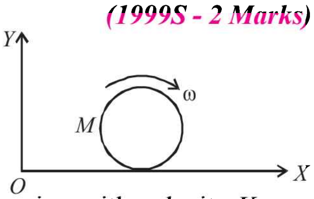
   (a) 0.42 *m* from mass of 0.3 kg
   (b) 0.70 *m* from mass of 0.7 kg
   (c) 0.98 *m* from mass of 0.3 kg
   (d) 0.98 *m* from mass of 0.7 kg

3. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity υ. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then *(1999S - 2 Marks)*

   (a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$
   (c) $\omega_A = \omega$ (d) $\omega_B = \omega$

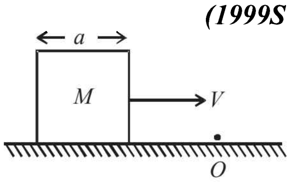
4. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Figure. The magnitude of angular momentum of the disc about the origin O is *(1999S - 2 Marks)*

   (a) $(1/2) MR^2\omega$
   (b) $MR^2\omega$
   (c) $(3/2) MR^2\omega$
   (d) $2 MR^2\omega$

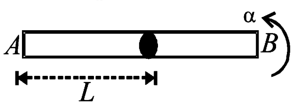


5. A cubical block of side a is moving with velocity V on a horizontal smooth plane as shown in Figure. It hits a ridge at point O. The angular speed of the block after it hits O is *(1999S - 2 Marks)*

   (a) $3V/(4a)$
   (b) $3V/(2a)$
   (c) $\sqrt{3V}/(\sqrt{2a})$
   (d) zero



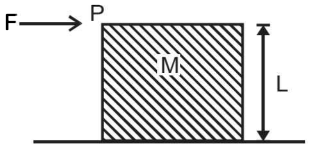
6. A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is *(2000S)*



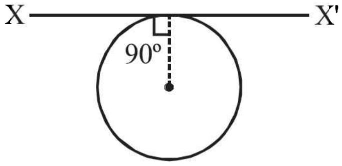
   (a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{\alpha}$ (c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

7. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is *(2000S)*

   (a) infinitesimal
   (b) mg/4
   (c) mg/2
   (d) $mg(1-\mu)$



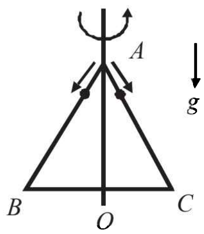
8. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is *(2000S)*



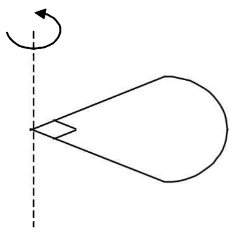
   (a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

9. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are *(2000S)*

   (a) angular velocity and total energy (kinetic and potential)
   (b) Total angular momentum and total energy
   (c) angular velocity and moment of inertia about the axis of rotation
   (d) total angular momentum and moment of inertia about the axis of rotation



10. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is *(2001S)*



   (a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$

   (c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}\ MR^2$

11. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are *(2002S)*

   (a) up the incline while ascending and down the incline descending
   (b) up the incline while ascending as well as descending
   (c) down the incline while ascending and up the incline while descending
   (d) down the incline while ascending as well as descending.

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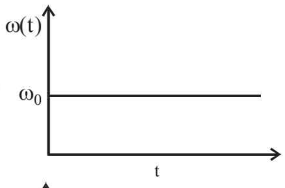
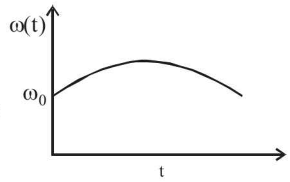
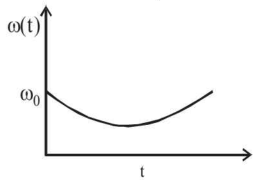
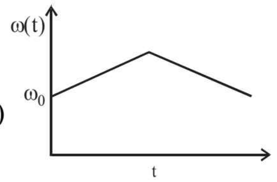
12. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as *(2002S)*

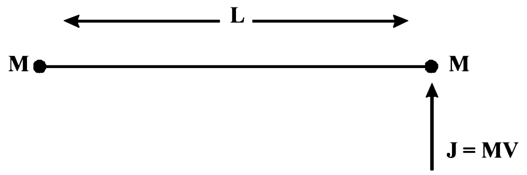
(a) 

(b) 

(c) 

(d) 

13. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J=MV$ is imparted to the body at one of its ends, what would be its angular velocity? *(2003S)*

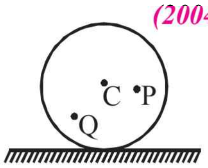


(a) V/L
(b) $2V/L$
(c) $V/3L$
(d) $V/4L$

14. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved? *(2003S)*
(a) centre of the circle
(b) on the circumference of the circle.
(c) inside the circle
(d) outside the circle.

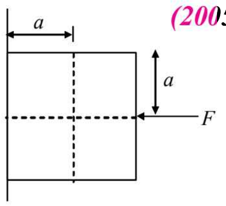
15. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be *(2004S)*
(a) $2K$ (b) $K/2$ (c) K (d) $K/4$

16. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is *(2004S)*



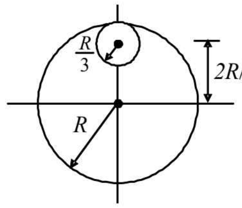
(a) $v_Q > v_C > v_P$
(b) $v_P > v_C > v_Q$
(c) $v_P = v_C, v_Q = v_C/2$
(d) $v_P < v_C > v_Q$

17. A block of mass m is at rest under the action of force F against a wall as shown in figure. Which of the following statement is incorrect? *(2005S)*



(a) $f = mg$ [f friction force]
(b) $F = N$ [N normal force]
(c) F will not produce torque
(d) N will not produce torque

18. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is *(2005S)*



(a) $4MR^2$
(b) $\frac{40}{9}MR^2$
(c) $10\,MR^2$
(d) $\frac{37}{9}MR^2$

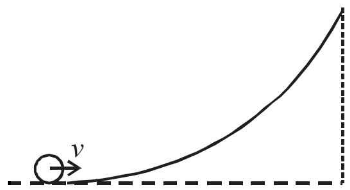
19. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct? *(2005S)*
(a) $\vec{L}$ (angular momentum) is conserved about the centre
(b) only direction of angular momentum $\vec{L}$ is conserved
(c) It spirals towards the centre
(d) its acceleration is towards the centre.

20. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as *(2006 - 3M, –1)*

(a) $r = \sqrt{\frac{2}{15}}R$ (b) $r = \frac{2}{\sqrt{15}}R$

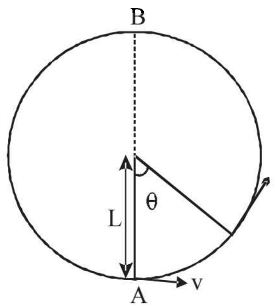
(c) $r = \frac{2}{15}R$ (d) $r = \frac{\sqrt{2}}{15}R$

21. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is *(2007)*



(a) ring (b) solid sphere
(c) hollow sphere (d) disc

22. A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies – *(2008)*



(a) $\theta = \frac{\pi}{4}$
(b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$
(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$
(d) $\frac{3\pi}{4} < \theta < \pi$

23. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6\,m$.

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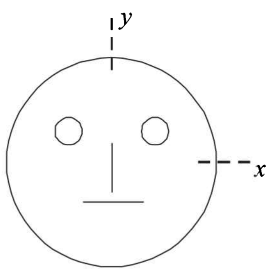
The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (– *a*, *a*), right inner circle (*a*, *a*), vertical line (0, 0) and horizontal line (0, – *a*). The *y*-coordinate of the centre of mass of the ink in this drawing is ***(2009)***



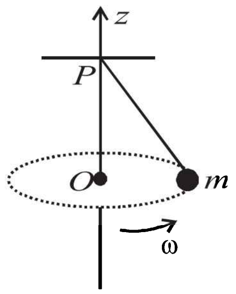
(a) $\frac{a}{10}$

(b) $\frac{a}{8}$

(c) $\frac{a}{12}$

(d) $\frac{a}{3}$

24. A small mass *m* is attached to a massless string whose other end is fixed at *P* as shown in the figure. The mass is undergoing circular motion in the *x*-*y* plane with centre at *O* and constant angular speed ω. If the angular momentum of the system, calculated about *O* and *P* are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



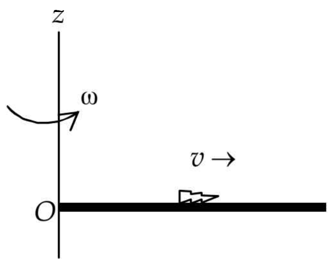
(a) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(b) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

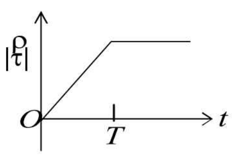
(c) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

(d) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

25. A thin uniform rod, pivoted at *O*, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time *t* = 0, a small insect starts from *O* and moves with constant speed *v*, with respect to the rod towards the other end. It reaches the end of the rod at *t* = *T* and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) about *O*, as a function of time is best represented by which plot? ***(2012)***



(a)



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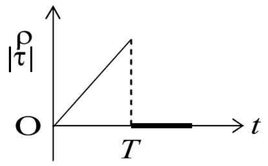

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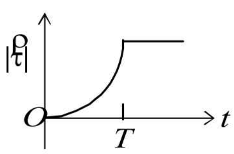
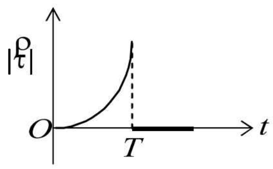


(d)



26. A uniform wooden stick of mass 1.6 kg and length *l* rests in an inclined manner on a smooth, vertical wall of height h(< *l*) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/*l* and the frictional force *f* at the bottom of the stick are (g = 10 m s^{-2}) ***(JEE Adv. 2016)***

(a) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}$N (b) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}$N

(c) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}$N (d) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}$N

D MCQ with One or More than One Correct

1. Two particles *A* and *B* initially at rest, move towards each other under mutual force of attraction. At the instant when the speed of *A* is *V* and the speed of *B* is 2*V*, the speed of the centre of mass of the system is ***(1982 - 3 Marks)***

(a) 3 *V* (b) *V*

(c) 1.5 *V* (d) zero

2. *A* mass *M* moving with a constant velocity parallel to the *X*-axis. Its angular momentum with respect to the origin ***(1985 - 2 Marks)***

(a) is zero (b) remains constant

(c) goes on increasing (d) goes on decreasing

3. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts ***(1990 - 2 Marks)***

(a) in the backward direction on the front wheel and in the forward direction on the rear wheel.

(b) in the forward direction on the front wheel and in the backward direction on the rear wheel.

(c) in the backward direction on both the front and the rear wheels.

(d) in the forward direction on both the front the rear wheels.

4. A particle of mass *m* is projected with a velocity *v* making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height *h* is ***(1990 - 2 Marks)***

(a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$

(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{2gh^3}$

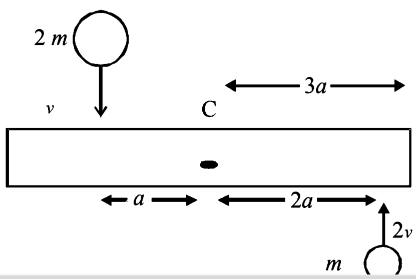
5. A uniform bar of length 6*a* and mass 8*m* lies on a smooth horizontal table. Two point masses *m* and 2*m* moving in the same horizontal plane with speed 2*v* and *v*, respectively, strike the bar [as shown in the fig.] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, *E* and v_c respectively, we have after collision ***(1991 - 2 Mark)***

(a) $v_c = 0$

(b) $\omega = \frac{3v}{5a}$

(c) $\omega = \frac{v}{5a}$

(d) $E = \frac{3mv^2}{5}$



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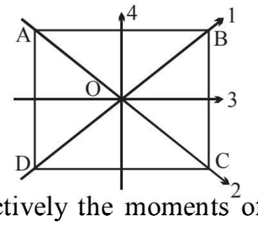
6. The moment of inertia of a thin square plate *ABCD*, Fig., of uniform thickness about an axis passing through the centre *O* and perpendicular to the plane of the plate is *(1992 - 2 Marks)*



(a) $I_1 + I_2$
(b) $I_3 + I_4$
(c) $I_1 + I_3$
(d) $I_1 + I_2 + I_3 + I_4$

where I_1, I_2, I_3 and I_4 are respectively the moments of intertial about axis 1, 2, 3 and 4 which are in the plane of the plate.

7. A tube of length *L* is filled completely with an incomressible liquid of mass *M* and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is *(1992 - 2 Marks)*

(a) $\frac{M\omega^2 L}{2}$ (b) $M\omega^2 L$
(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$

8. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A pendulum bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with track is *(1992 - 2 Mark)*

(a) zero (b) 30°
(c) 45° (d) 60°

9. Let *I* be the moment of inertia of a uniform square plate about an axis *AB* that passes through its centre and is parallel to two of its sides. *CD* is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with *AB*. The moment of inertia of the plate about the axis *CD* is then equal to *(1998S - 2 Marks)*

(a) I (b) $I\sin^2\theta$
(c) $I\cos^2\theta$ (d) $I\cos^2(\theta/2)$

10. The torque τ on a body about a given point is found to be equal to $\boldsymbol{A} \times \boldsymbol{L}$ where $\boldsymbol{A}$ is a constant vector, and $\boldsymbol{L}$ is the angular momentum of the body about that point. From this it follows that *(1998S - 2 Marks)*

(a) $\frac{d\boldsymbol{L}}{dt}$ is perpendicular to $\boldsymbol{L}$ at all instants of time.
(b) the component of $\boldsymbol{L}$ in the direction of $\boldsymbol{A}$ does not change with time.
(c) the magnitude of $\boldsymbol{L}$ does not change with time.
(d) $\boldsymbol{L}$ does not change with time

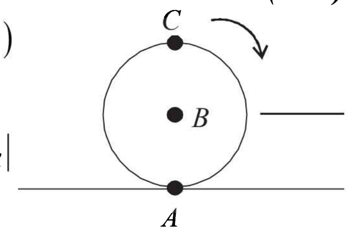
11. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then *(2006 - 5M, –1)*

(a) The friction force is dissipative
(b) The friction force is necessarily changing
(c) The friction force will aid rotation but hinder translation
(d) The friction force is reduced if θ is reduced

12. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that *(2009)*

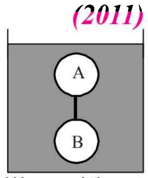
(a) linear momentum of the system does not change in time
(b) kinetic energy of the system does not change in time
(c) angular momentum of the system does not change in time
(d) potential energy of the system does not change in time

13. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, *(2009)*



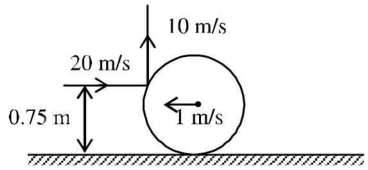
(a) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$
(b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
(c) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$
(d) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

14. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if *(2011)*



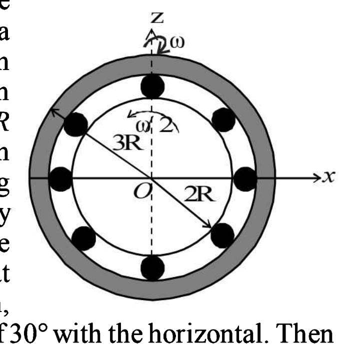
(a) $d_A < d_F$
(b) $d_B > d_F$
(c) $d_A > d_F$
(d) $d_A + d_B = 2d_F$

15. A thin ring of mass 2 kg and radius 0.5 m is rolling without on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision *(2011)*



(a) the ring has pure rotation about its stationary CM.
(b) the ring comes to a complete stop.
(c) friction between the ring and the ground is to the left.
(d) there is no friction between the ring and the ground.

16. The figure shows a system consisting of (i) a ring of outer radius 3*R* rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2*R* rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The point *P* on the inner disc is at a distance *R* from the origin, where *OP* makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, *(2012)*



(a) the point *O* has linear velocity $3R\omega\,\hat{i}$
(b) the point *P* has linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$.
(c) the point *P* has linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$
(d) the point *P* has linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

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17. Two solid cylinders *P* and *Q* of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder *P* has most of its mass concentrated near its surface, while *Q* has most of its mass concentrated near the axis. Which statement(s) is(are) correct? *(2012)*

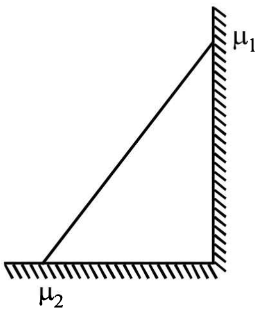
(a) Both cylinders *P* and *Q* reach the ground at the same time.

(b) Cylinders *P* has larger linear acceleration than cylinder *Q*.

(c) Both cylinders reach the ground with same translational kinetic energy.

(d) Cylinder *Q* reaches the ground with larger angular speed.

18. In the figure, a ladder of mass *m* is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then *(JEE Adv. 2014)*



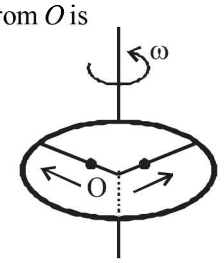
(a) $\mu_1 = 0, \mu_2 \neq 0$ and $N_2 \tan\theta = \dfrac{mg}{2}$

(b) $\mu_1 \neq 0, \mu_2 = 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

(c) $\mu_1 \neq 0, \mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(d) $\mu_1 = 0, \mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

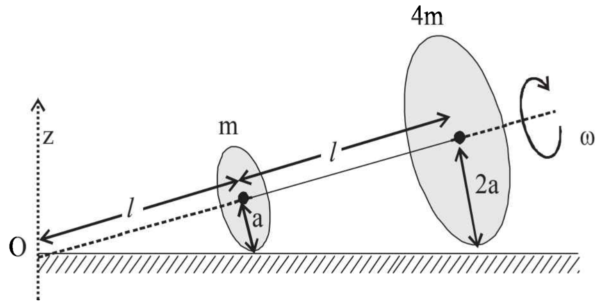
19. A ring of mass *M* and radius *R* is rotating with angular speed ω about a fixed vertical axis passing through its centre *O* with two point masses each of mass $\dfrac{M}{8}$ at rest at *O*. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\dfrac{8}{9}\omega$ and one of the masses is at a distance of $\dfrac{3}{5}R$ from *O*. At this instant the distance of the other mass from *O* is *(JEE Adv. 2015)*



(a) $\dfrac{2}{3}R$ (b) $\dfrac{1}{3}R$

(c) $\dfrac{3}{5}R$ (d) $\dfrac{4}{5}R$

20. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24a}$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is (are) true? *(JEE Adv. 2016)*



(a) The centre of mass of the assembly rotates about the z-axis with an angular speed of ω/5

(b) The magnitude of angular momentum of center of mass of the assembly about the point O is 81 $ma^2\omega$

(c) The magnitude of angular momentum of the assembly about its center of mass is 17 $ma^2\omega/2$.

(d) The magnitude of the z-component of $\vec{L}$ is 55 $ma^2\omega$.

21. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j},$$

where $\alpha = 10/3\ ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? *(JEE Adv. 2016)*

(a) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\ ms^{-1}$

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -5/3)\ \hat{k}$ N m s

(c) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\ \hat{k}$ N m

E Subjective Problems

1. A 40 kg mass, hanging at the end of a rope of length *l*, oscillates in a vertical plane with an angular amplitude θ_0. What is the tension in the rope when it makes an engle θ with the vertical? If the breaking strength of the rope is 80 kg, what is the maximum amplitude with which the mass can oscillate without the rope breaking? *(1978)*

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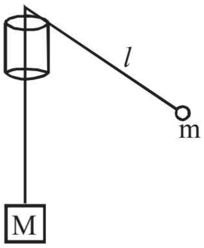
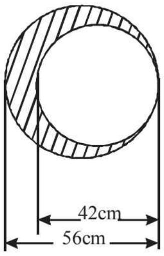
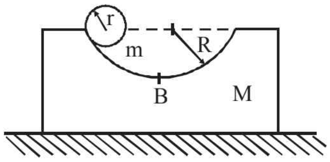
2. A large mass M and a small mass m hang at two ends of a string that passes over a smooth tube as shown in the figure. The mass m moves around a circular path which lies in a horizontal plane. The length of string from the mass m to the top of the tube is l and θ is the 'angle' this length makes with the vertical. What should be the frequency of rotation of mass m, so that the mass M remains stationary? *(1978)*



3. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in figure. Find the position of the centre of mass of the remaining portion. *(1980)*

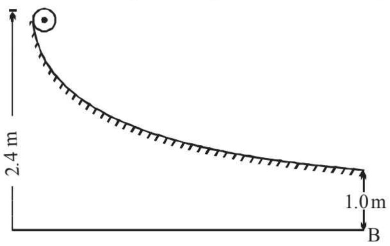


4. A block of mass M with a semicircular of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track ? *(1983 - 7 Marks)*

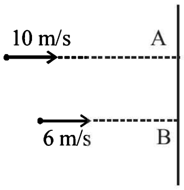


5. A particle is projected at time $t = 0$ from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = v_0/g$ *(1984- 6 Marks)*

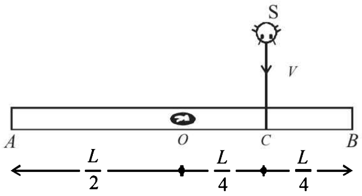
6. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part, The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in fig.) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass? Explain. *(1987 - 7 Marks)*



7. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length $\sqrt{3}$ meters. Two particles, each of mass 0.08 kg, are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 m/s, and other with 6 m/s as shown in fig. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of 0.5m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of the kinetic energy of the system in the above collision process. *(1989 - 8 Marks)*

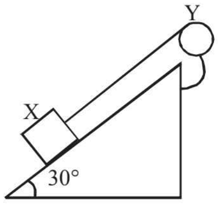


8. A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (Fig). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω. *(1992 - 8 Marks)*
   (a) Determine the angular velocity ω in terms of V and L.
   (b) If the insect reaches the end B when the rod has turned through an angle of 90°, determine V.

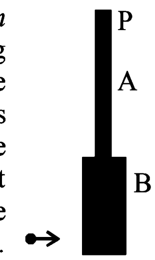


9. A uniform thin rod of mass M and length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling. *(1993-1+5 Marks)*
   (i) What is the path followed by the centre of mass of the rod during its fall?
   (ii) Find the equation to the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point?

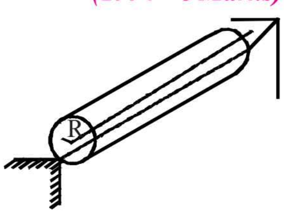
10. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in Figure. The drum is given an initial angular velocity such that the block X starts moving up the plane. *(1994 - 6 Marks)*
   (i) Find the tension in the string during the motion.
   (ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1} calculate the distance travelled by X from that instant of time until it comes to rest



11. Two uniform thin rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in fig. Such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position. *(1994 - 6 Marks)*



12. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest its length parallel to the edge such that the axis of the



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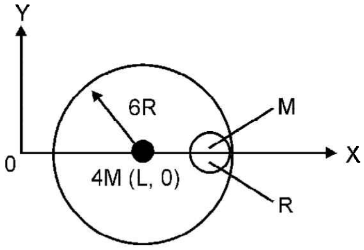
cylinder and the edge of the block are in the same vertical plane as shown in the figure below. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine: *(1995 - 10 Marks)*

(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge,

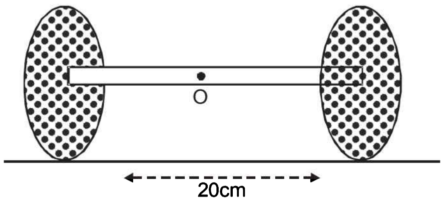
(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and

(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

13. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$ (Fig. P-3). The masses of large and small spheres are $4M$ and M, respectively, This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position. *(1996 - 3 Marks)*



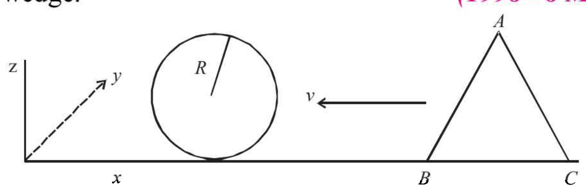
14. Two thin circular disks of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disk through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of the motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x axis as the direction of motion of the truck and z axis as the vertically upwards direction. If the truck has an acceleration of 9 m/s^2. Calculate: *(1997 - 5 Marks)*



(i) The force of friction on each disk,

(ii) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ in the x, y, and z directions.

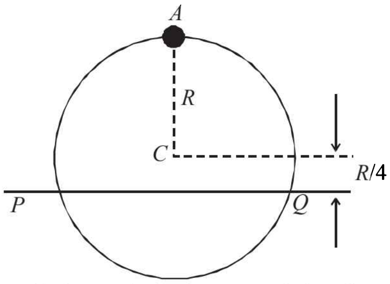
15. A wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $-v\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in Figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time. Δt, during which the sphere exerts a constant force F on the wedge. *(1998 - 8 Marks)*



(a) Find the force F and also the normal force N exerted by the table on the wedge during the time Δt.

(b) Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of F. Find the magnitude of the torque due to the normal force N about the centre of the wedge, during the interval Δt.

16. A uniform circular disc has radius R and mass m. A particle also of mass m, is fixed at a point A on the edge of the disc as shown in Figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ.



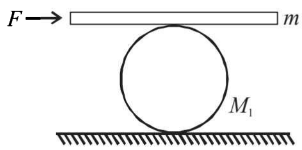
Initially, the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position. *(1998 - 8 Marks)*

17. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown in Figure. There in no slipping at any contact. The horizontal component of the force applied by the man is F. *(1999 - 10 Marks)*

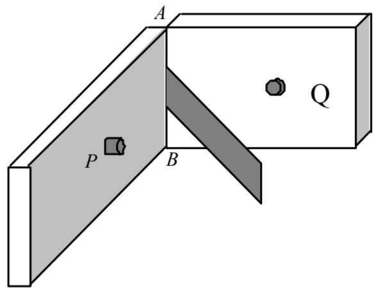
Find

(a) the accelerations of the plank and the center of mass of the cylinder, and

(b) the magnitudes and directions of frictional forces at contact points.



18. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its center of mass is 1.2 kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. *(2001-10 Marks)*



Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s,

(a) Find the location of the center of mass of the laminar sheet from AB.

(b) At what angular velocity does the laminar sheet come back after the first impact?

(c) After how many impacts, does the laminar sheet come to rest?

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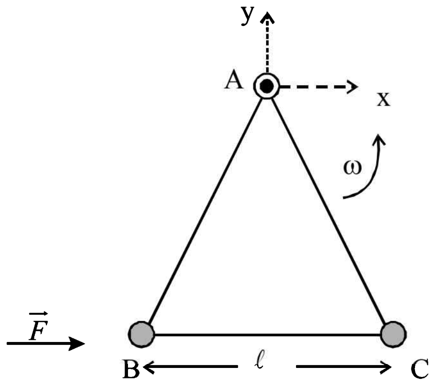
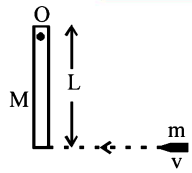
19. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictioness table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. *(2002 - 5 Marks)*



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T.

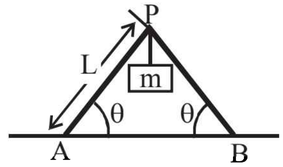
20. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O.



(2005 - 2 Marks)

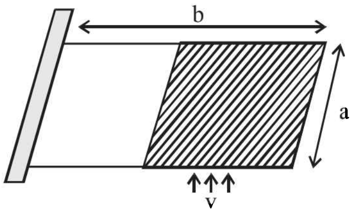
21. A cylinder of mass m and radius R rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder. *(2005 - 4 Marks)*

22. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.



(2005 - 4 Marks)

23. A rectangular plate of mass M and dimension $a \times b$ is held in horizontal position by striking n small balls (each of mass m) per unit area per second. The balls are striking in the shaded half region of the plate. The collision of the balls with the plate is elastic. What is v? *(2006 - 6M)*

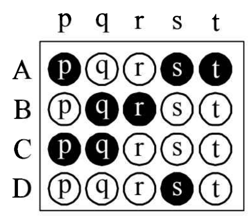


(Given $n = 100$, $M = 3$ kg, $m = 0.01$ kg; $b = 2$ m; $a = 1$ m; $g = 10$ m/s^2).

F Match the Following

DIRECTIONS (Q. No. 1) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. **Column-II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column-I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II**. *(2009)*

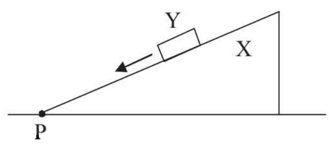
Column-I

(A) The force exerted by X on Y has a magnitude Mg.

(B) The gravitational potential energy of X is continuously increasing.

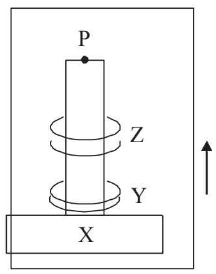
Column II

(p)



Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

(q)



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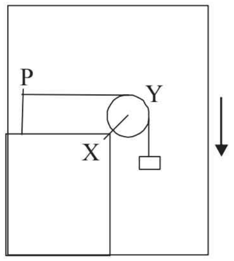
Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(C) Mechanical energy of the system X + Y is continuously decreasing.

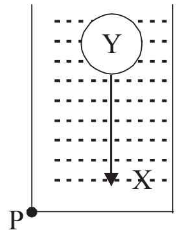
(r)



A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.

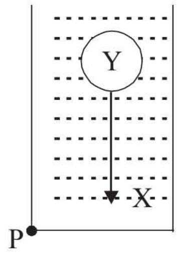
(D) The torque of the weight of Y about point P is zero.

(s)



A sphere Y of mass M is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

(t)



A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

G Comprehension Based Questions

PASSAGE - 1

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

1. The ratio x_1/x_2 is *(2007)*

(a) 2 (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

2. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is *(2007)*

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$ (c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

3. The loss of kinetic energy in the above process is *(2007)*

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$

(c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$

PASSAGE - 2

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached

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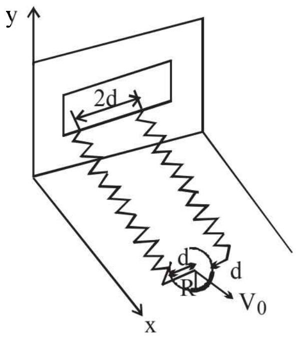


to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in horizontal plane.
The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. ***(2008)***

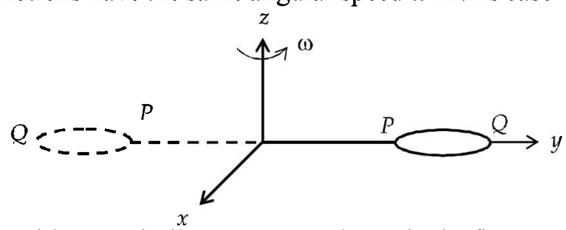
4. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is
(a) −kx (b) −2kx (c) −2kx/3 (d) −4kx/3

5. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to –
(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$ (c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

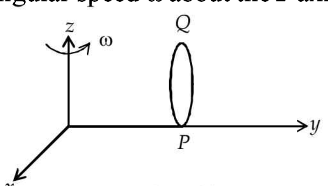
6. The maximum value of V_0 for which the disk will roll without slipping is –
(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$ (c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

PASSAGE-3

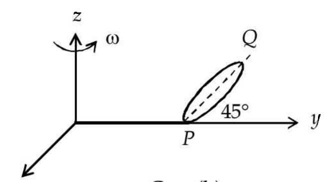
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass.
These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless, stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the *z*-axis and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points *P* and *Q*). Both these motions have the same angular speed ω in this case ***(2012)***



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to *x-z* plane; Case (b) the disc with its face making an angle of 45° with *x-y* plane and its horizontal diameter parallel to *x*-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the *z*-axis.



Case (a)



Case (b)

7. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
(a) It is vertical for both the cases (a) and (b)
(b) It is vertical for case (a); and is at 45° to the *x-z* plane and lies in the plane of the disc for case (b).
(c) It is horizontal for case (a); and is at 45° to the *x-z* plane and is normal to the plane of the disc for case (b).
(d) It is vertical for case (a); and is 45° to the *x-z* plane and is normal to the plane of the disc for case (b).

8. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?
(a) It is $\sqrt{2}\omega$ for both the cases
(b) It is ω for case (a); and $\omega/\sqrt{2}$ for case (b)
(c) It is ω for case (a); and $\sqrt{2\omega}$ for case (b)
(d) It is ω for both the cases.

H Assertion & Reason Type Questions

1. **STATEMENT-1:** If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.
STATEMENT-2: The linear momentum of an isolated system remains constant. ***(2007)***
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

2. **STATEMENT-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.
STATEMENT-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. ***(2008)***
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

I Integer Value Correct Type

1. A binary star consists of two stars A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is ***(2010)***

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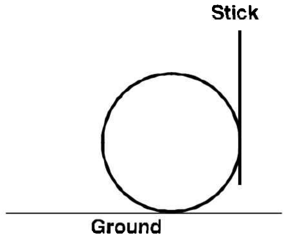
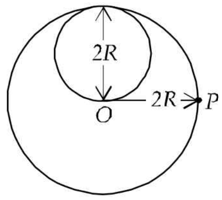
2. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is *(2011)*



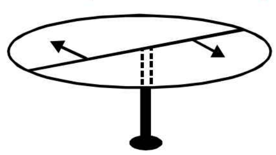
3. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg– m^2, then N is. *(2011)*

4. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is *(2012)*

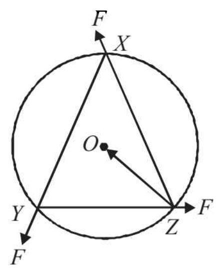


5. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is *(JEE Adv. 2013)*

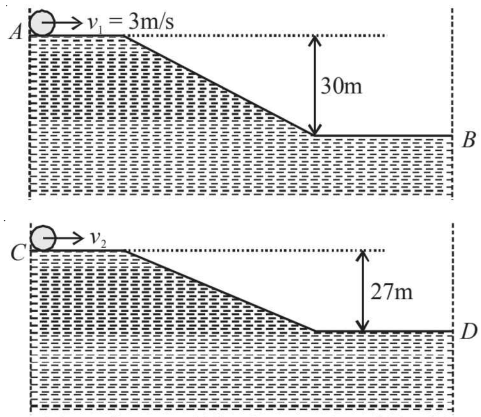
6. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is *(JEE Adv. 2014)*



7. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is *(JEE Adv. 2014)*



8. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s then v_2 in m/s is ($g = 10$ m/s^2) *(JEE Adv. 2015)*



9. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If, $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc? [2002]

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$.

2. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is [2002]

(a) 60 (b) 30 (c) 15 (d) 25

3. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is [2002]

(a) 10 (b) 20 (c) 25.5 (d) 5

4. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is [2002]

(a) v (b) $v/3$ (c) $v/2$ (d) zero.

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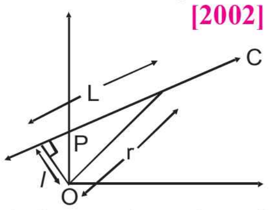
5. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling) **[2002]**
(a) solid sphere (b) hollow sphere
(c) ring (d) all same.

6. Moment of inertia of a circular wire of mass M and radius R about its diameter is **[2002]**
(a) $MR^2/2$ (b) MR^2 (c) $2MR^2$ (d) $MR^2/4$.

7. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P? **[2002]**



(a) mvL
(b) mvl
(c) mvr
(d) zero.

8. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is **[2003]**
(a) $I_Y = 32 I_X$ (b) $I_Y = 16 I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64 I_X$

9. A particle performing uniform circular motion has angular frequency is doubled & its kinetic energy halved, then the new angular momentum is **[2003]**
(a) $\frac{L}{4}$ (b) $2L$ (c) $4L$ (d) $\frac{L}{2}$

10. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$, and $\vec{T}$ be the torque of this force about the origin. Then **[2003]**
(a) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} \neq 0$ (b) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} = 0$
(c) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} \neq 0$ (d) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} = 0$

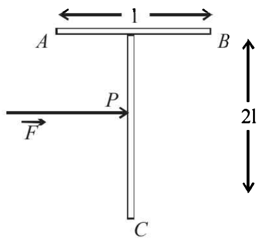
11. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected? **[2004]**
(a) Angular velocity (b) Angular momentum
(c) Moment of inertia (d) Rotational kinetic energy

12. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B Such that **[2004]**
(a) $I_A < I_B$ (b) $I_A > I_B$
(c) $I_A = I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$
where d_A and d_B are their densities.

13. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards **[2005]**
(a) does not shift
(b) depends on height of breaking
(c) body B
(d) body C

14. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[2005]**
(a) $\frac{2}{5}Mr^2$ (b) $\frac{1}{4}Mr$ (c) $\frac{1}{2}Mr^2$ (d) Mr^2

15. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[2005]**

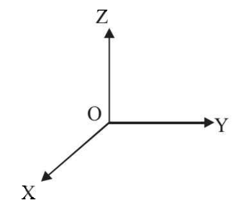


(a) $\frac{3}{2}\ell$ (b) $\frac{2}{3}\ell$ (c) ℓ (d) $\frac{4}{3}\ell$

16. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle is moved, so as to keep the centre of mass at the same position? **[2006]**
(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$ (c) $\frac{m_1}{m_2}d$ (d) d

17. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is **[2006]**
(a) $2m\ell^2$ (b) $\sqrt{3}m\ell^2$ (c) $3m\ell^2$ (d) $m\ell^2$

18. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is **[2006]**



(a) $F(\hat{i} - \hat{j})$
(b) $-F(\hat{i} + \hat{j})$
(c) $F(\hat{i} + \hat{j})$
(d) $-F(\hat{i} - \hat{j})$

19. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$ **[2006]**
(a) $\frac{\omega(m+2M)}{m}$ (b) $\frac{\omega(m-2M)}{(m+2M)}$
(c) $\frac{\omega m}{(m+M)}$ (d) $\frac{\omega m}{(m+2M)}$

20. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α / R form the centre of the bigger disc. The value of α is **[2007]**
(a) 1/4 (b) 1/3 (c) 1/2 (d) 1/6

21. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[2007]**

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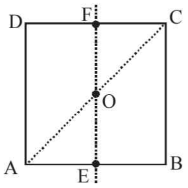
(a) $\dfrac{g\sin\theta}{1-MR^2/I}$ (b) $\dfrac{g\sin\theta}{1+I/MR^2}$

(c) $\dfrac{g\sin\theta}{1+MR^2/I}$ (d) $\dfrac{g\sin\theta}{1-I/MR^2}$

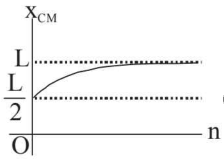
22. Angular momentum of the particle rotating with a central force is constant due to **[2007]**
(a) constant torque
(b) constant force
(c) constant linear momentum
(d) zero torque

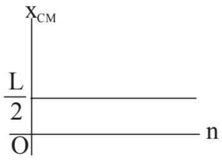
23. For the given uniform square lamina $ABCD$, whose centre is O, **[2007]**

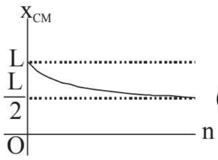


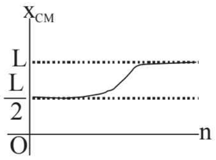
(a) $I_{AC} = \sqrt{2}\, I_{EF}$
(b) $\sqrt{2} I_{AC} = I_{EF}$
(c) $I_{AD} = 3I_{EF}$
(d) $I_{AC} = I_{EF}$

24. A thin rod of length 'L' is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\dfrac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n? **[2008]**

(a)


(b)


(c)


(d)


25. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[2008]**

(a) $\dfrac{5}{6}ma^2$ (b) $\dfrac{1}{12}ma^2$ (c) $\dfrac{7}{12}ma^2$ (d) $\dfrac{2}{3}ma^2$

26. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of: **[2009]**

(a) $\dfrac{1}{6}\dfrac{l\omega}{g}$ (b) $\dfrac{1}{2}\dfrac{l^2\omega^2}{g}$ (c) $\dfrac{1}{6}\dfrac{l^2\omega^2}{g}$ (d) $\dfrac{1}{3}\dfrac{l^2\omega^2}{g}$

27. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is: **[2011]**

(a) g (b) $\dfrac{2}{3}g$ (c) $\dfrac{g}{3}$ (d) $\dfrac{3}{2}g$

28. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc.
(a) continuously decreases **[2011]**
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

29. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 the number of rotations made by the pulley before its direction of motion is reversed, is: **[2011]**
(a) more than 3 but less than 6
(b) more than 6 but less than 9
(c) more than 9
(d) less than 3

30. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ? **[JEE main 2013]**

(a) $\dfrac{r\omega_0}{4}$ (b) $\dfrac{r\omega_0}{3}$ (c) $\dfrac{r\omega_0}{2}$ (d) $r\omega_0$

31. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension: **[JEE Main 2014]**
(a) angular momentum is conserved.
(b) angular momentum changes in magnitude but not in direction.
(c) angular momentum changes in direction but not in magnitude.
(d) angular momentum changes both in direction and magnitude.

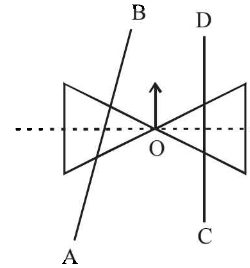
32. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : **[JEE Main 2015]**

(a) $\dfrac{5h}{8}$ (b) $\dfrac{3h^2}{8R}$ (c) $\dfrac{h^2}{4R}$ (d) $\dfrac{3h}{4}$

33. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is : **[JEE Main 2015]**

(a) $\dfrac{4MR^2}{9\sqrt{3}\pi}$ (b) $\dfrac{4MR^2}{3\sqrt{3}\pi}$ (c) $\dfrac{MR^2}{32\sqrt{2}\pi}$ (d) $\dfrac{MR^2}{16\sqrt{2}\pi}$

34. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and Cd (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :



(a) go straight. **[JEE Main 2016]**
(b) turn left and right alternately.
(c) turn left.
(d) turn right.

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6 Rotational Motion

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{2}{3}mg$ 2. $\frac{M\omega_0}{M+6m}$ 3. $\frac{1}{3}MRA\omega^2$ 4. –3

5. $\left(\frac{d-x}{d}\right)W, \frac{xW}{d}$ 6. 4.8 Ma^2

B 1. F 2. F 3. F 4. F

C 1. (c) 2. (c) 3. (c) 4. (c) 5. (a) 6. (a) 7. (c) 8. (d)
9. (b) 10. (a) 11. (b) 12. (b) 13. (a) 14. (a) 15. (b) 16. (b) 17. (d)
18. (a) 19. (b) 20. (b) 21. (d) 22. (d) 23. (a) 24. (c) 25. (b) 26. (d)

D 1. (d) 2. (b) 3. (a,c) 4. (b, d) 5. (a, c, d) 6. (a, b, c) 7. (a) 8. (c)
9. (a) 10. (a, b, c) 11. (c, d) 12. (a) 13. (b,c) 14. (a, b, c) 15. (c) 16. (a, b) 17. (d)
18. (c, d) 19. (d) 20. (a,c) 21. (a,b)

E 1. $T = mg[3\cos\theta - 2\cos\theta_0], \theta_0 = 30°$ 2. $\nu = \frac{1}{2\pi}\sqrt{\frac{gM}{\ell m}}$

3. 9 cm from the origin towards left 4. $\frac{m(R-r)}{M+m}, m\sqrt{\frac{2g(R-r)}{M(m+M)}}$

5. $\frac{mv_0^3}{2\sqrt{2}g}$, perpendicular to the plane of motion and is directed away from the reader. 6. 2m, yes

7. 2.72 J 8. (a) $\frac{12v}{7L}$ (b) 3.5 ms^{-1}

9. (i) Straight line, (ii) $\left[\frac{x}{\frac{L}{2}-r}\right]^2 + \left(\frac{y}{r}\right)^2 = 1$, Ellipse 10. (i) 1.63N (ii) 1.22 m 11. 6.3 m/s

12. (a) $\theta = \cos^{-1}\frac{4}{7}$ (b) $\sqrt{\frac{4gR}{7}}$ (c) 6 13. $(L+2R, 0)$

14. (i) $6\hat{i}$ (ii) $0.6(\hat{k}-\hat{j}), 0.6(-\hat{j}-\hat{k})$

15. (a) $\frac{2mv}{\sqrt{3}\Delta t}(\sqrt{3}\hat{i}-\hat{k}), \left(\frac{2mv}{\sqrt{3}\Delta t}+mg\right)\hat{k}$ (b) $\frac{4mv}{\sqrt{3}\Delta t}\times h$ 16. $\sqrt{5gR}$

17. (a) $\frac{8F}{3M_1+8m_2}, \frac{4F}{3M_1+8m_2}$ (b) $\frac{3FM_1}{3M_1+8m_2}, \frac{FM_1}{3M_1+8m_2}$

18. (a) 0.1 m (b) 1 radian/sec. (c) infinite 19. (a) $\sqrt{3}m\ell\omega^2$ (b) $(F_{net})_x = -\frac{F}{4}, (F_{net})_y = \sqrt{3}m\ell\omega^2$

20. $\omega = \frac{3mv}{(M+3m)L}$ 21. $a = \frac{2}{3}g\sin\theta$

22. $\left[(M+m)\frac{g}{2}\right]\cot\theta$, along AB. 23. 10 m/s

F 1. (A)→ p, t; (B)→ q, s, t; (C)→ p, r, t; (D)→ q, p

G 1. (c) 2. (a) 3. (b) 4. (d) 5. (d) 6. (c) 7. (a) 8. (b)

H 1. (d) 2. (d)

I 1. 6 2. 4 3. 9 4. 3 5. 8 6. 4 7. 2 8. 7 9. 6

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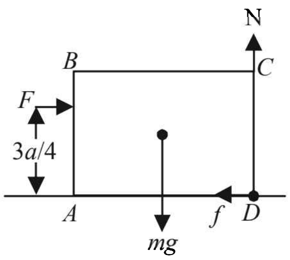
Section-B : JEE Main/ AIEEE

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (a)
10. (d)	**11.** (b)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)
19. (d)	**20.** (b)	**21.** (b)	**22.** (d)	**23.** (d)	**24.** (a)	**25.** (d)	**26.** (c)	**27.** (b)
28. (c)	**29.** (a)	**30.** (c)	**31.** (c)	**32.** (d)	**33.** (a)	**34.** (c)		

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. **KEY CONCEPT**
When the cube begins to tip about the edge the normal reaction will pass through the edge about which rotation takes place. The torque due to N and f will be zero.
Taking moment of force about D



$$F\times\frac{3a}{4}=mg\times\frac{a}{2}\quad\therefore\quad F=\frac{2}{3}mg$$

2. **Note :** Since no external force and hence no torque is applied, the angular momentum remains constant

$\therefore\quad I_1\omega_1=I_2\omega_2$

$$\therefore\quad \omega_2=\frac{I_1\omega_1}{I_2}=\frac{\frac{ML^2}{12}\times\omega_0}{\frac{ML^2}{12}+2m\times\left(\frac{L}{2}\right)^2}=\frac{M\omega_0}{M+6m}$$

3. Considering the motion of the platform
$x=A\cos\omega t$

$$\Rightarrow\ \frac{dx}{dt}=-A\omega\sin\omega t\ \Rightarrow\ \frac{d^2x}{dt^2}=-A\omega^2\cos\omega t$$

The magnitude of the maximum acceleration of the platform is

$\therefore\quad |\text{Max acceleration}|=A\omega^2$

When platform moves a torque acts on the cylinder and the cylinder rotates about its axis.

Acceleration of cylinder, $a_1=\dfrac{f}{m}$

Torque $\tau=fR\quad\therefore\quad I\alpha=fR$

$$\alpha=\frac{fR}{I}=\frac{fR}{MR^2/2}$$

or, $$\alpha=\frac{2f}{MR}\quad\text{or}\quad R\alpha=\frac{2f}{M}$$

$\therefore$ Equivalent linear acceleration $(R\alpha=a_2)$

$$a_2=\frac{2f}{M}$$

$\therefore$ Total linear acceleration,

$$a_{max}=a_1+a_2=\frac{f}{M}+\frac{2f}{M}=\frac{3f}{M}$$

or, $A\omega^2=\dfrac{3f}{M}$ or, $f=\dfrac{MA\omega^2}{3}$

Thus, maximum torque,

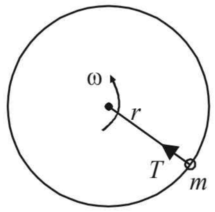
$$\tau_{max}=f\times R=\frac{MA\omega^2R}{3}=\frac{1}{3}MAR\omega^2$$

4. Let at any instant of time t, the radius of the horizontal surface be r.

$T=mr\omega^2$ ... (i)

Where m is the mass of stone and ω is the angular velocity at that instant of time t.

Also, $L=I\omega$ ... (ii)



From (i) and (ii)

$$T=\frac{mrL^2}{I^2}=\frac{mL^2}{(mr^2)^2}\times r,\quad T=\left(\frac{L^2}{m}\right)r^{-3}$$

$$=Ar^{-3}\qquad\left(\text{where }\frac{L^2}{m}=A\text{ is constant}\right)$$

Thus, $n=-3$

5. $R_A+R_B=W$

$\therefore R_A=W-R_B$

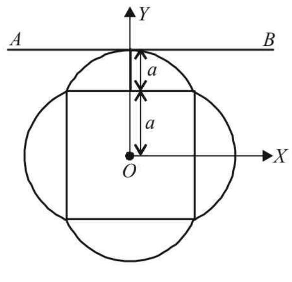
6. Assuming symmetric lamina to be in xy plane, we will have $I_x=I_y$ (Since the mass distribution is same about x-axis and y-axis)

$I_x+I_y=I_z$ (perpendicular-axis theorem)

It is given that $I_z=1.6\,Ma^2$.

Hence

$$I_x=I_y=\frac{I_z}{2}=0.8\,Ma^2$$



Now, according to parallel-axis theorem, we get

$I_{AB}=I_x+M(2a)^2$
$=0.8\,Ma^2+4Ma^2$
$=4.8\,Ma^2$

B. True/False

1. $\tau=I\alpha\quad\therefore\alpha=\dfrac{\tau}{I}$

$\tau=$ Force $\times$ perpendicular distance. Torque is same in both the cases. But since, I will be different due to different mass distribution about the axis,

$\therefore$ α will be different.

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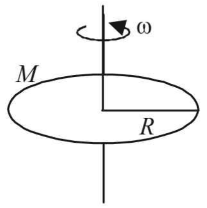
2. $\vec{\tau} = \frac{d\vec{L}}{dt}$ Since, $\vec{\tau} = 0$



∴ $\vec{L}$ = constant

∴ $I_1\omega_1 = I_2\omega_2$

$I_1 = \frac{1}{2}MR^2$

$\omega_1 = \omega$

$$I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\frac{M}{4}R^2 = \left(\frac{4+1}{8}\right)MR^2 = \frac{5}{8}MR^2$$

$\omega_2 = ?$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = +\frac{\frac{1}{2}MR^2 \times \omega}{\frac{5}{8}MR^2} = \frac{8}{2\times 5}\omega = \frac{4}{5}\omega$$

3. Total energy of the ring

$= (K.E.)_{Rotation} + (K.E.)_{Translational}$

$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_c^2$

$= \frac{1}{2}\times mr^2\omega^2 + \frac{1}{2}m(r\omega)^2 \quad (\because I = mr^2,\ v_c = r\omega)$

$= mr^2\omega^2$

Total kinetic energy of the cylinder

$= (K.E.)_{Rotation} + (K.E.)_{Translational}$

$= \frac{1}{2}I'\omega'^2 + \frac{1}{2}Mv'^2_c$

$= \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega'^2 + \frac{1}{2}M(r\omega')^2$

$= \frac{3}{4}Mr^2\omega'^2$ ... (i)

Equating (i) and (ii)

$mr^2\omega^2 = \frac{3}{4}Mr^2\omega'^2$

$\Rightarrow \frac{\omega'^2}{\omega^2} = \frac{4m}{3M} = \frac{4}{3}\times\frac{0.3}{0.4} = 1$

$\Rightarrow \omega' = \omega$

Both will reach at the same time.

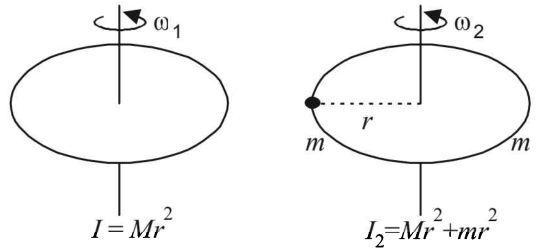
4. Since no external force is acting on the two particle system

∴ $a_{c.m} = 0$

$\Rightarrow V_{c.m}$ = Constant.

C. MCQs with ONE Correct Answer

1. **(c)** Since the objects are placed gently, therefore no external torque is acting on the system. Hence angular momentum is constant.



i.e., $I_1\omega_1 = I_2\omega_2$

$Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\,\omega_2 \quad (\because \omega_1 = \omega)$

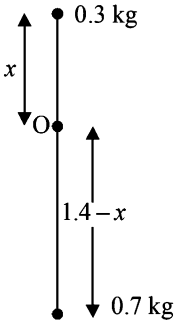
∴ $\omega_2 = \frac{M\omega}{M+2m}$

2. **(c)** The moment of inertia of the system about axis of rotation O is

$I = I_1 + I_2 = 0.3x^2 + 0.7\,(1.4 - x)^2$

$= 0.3x^2 + 0.7\,(1.96 + x^2 - 2.8x)$

$= x^2 + 1.372 - 1.96x$



The work done in rotating the rod is converted into its rotational kinetic energy.

∴ $W = \frac{1}{2}I\omega^2$

$= \frac{1}{2}[x^2 + 1.372 - 1.96x]\,\omega^2$

For work done to be minimum

$\frac{dW}{dx} = 0 \quad \Rightarrow 2x - 1.96 = 0$

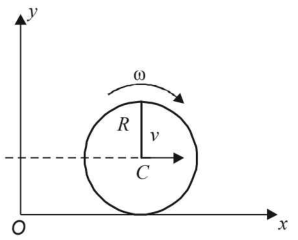
$\Rightarrow x = \frac{1.96}{2} = 0.98\,\text{m}$

3. **(c)** As the spheres are smooth there will be no friction (no torque) and therefore there will be no transfer of angular momentum. Thus A, after collision will remain with its initial angular momentum. i.e., $\omega_A = \omega$

4. **(c)** **KEY CONCEPT**

The disc has two types of motion namely translational and rotational. Therefore there are two types of angular momentum and the total angular momentum is the vector sum of these two.

In this case both the angular momentum have the same direction (perpendicular to the plane of paper and away from the reader).



$\vec{L} = \vec{L}_T + \vec{L}_R$

L_T = angular momentum due to translational motion.

L_R = angular momentum due to rotational motion about $C.M.$

$L = MV \times R + I_{cm}\omega$

$I_{cm} = M.I.$ about centre of mass C.

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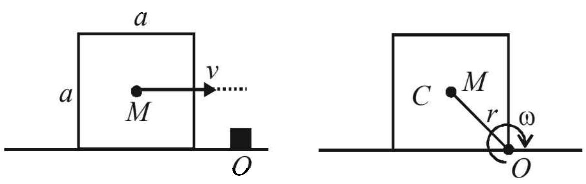
$= M(R\omega)R + \frac{1}{2}MR^2\omega$

($v = R\omega$ in case of rolling motion and surface at rest)

$= \frac{3}{2}MR^2\omega$

5. (a) 

$r = \sqrt{2}\frac{a}{2}$ or $r^2 = \frac{a^2}{2}$

Net torque about O is zero.

Therefore, angular momentum (L) about O will be conserved, or $L_i = L_f$

$$MV\left(\frac{a}{2}\right) = I_0\omega = (I_{cm} + Mr^2)\omega$$

$$\omega = \left\{\frac{Ma^2}{6} + M\left(\frac{a^2}{2}\right)\right\}\omega = \frac{2}{3}Ma^2\omega = \frac{3v}{4a}$$

6. (a) **Note :** When we are giving an angular acceleration to the rod, the bead is also having an instantaneous acceleration $a = L\alpha$. This will happen when a force is exerted on the bead by the rod. The bead has a tendency to move away from the centre. But due to the friction between the bead and the rod, this does not happen to the extent to which frictional force is capable of holding the bead.

The frictional force here provides the necessary centripetal force. If instantaneous angular velocity is ω then

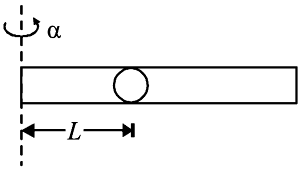
$mL\omega^2 = \mu(ma) \Rightarrow mL\omega^2 = \mu\, mL\alpha \Rightarrow \omega^2 = \mu\alpha$

By applying

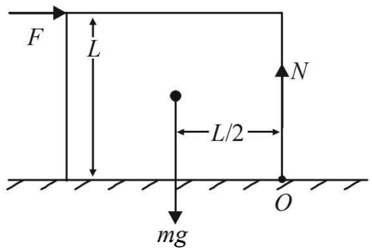
$\Rightarrow \quad \omega = \omega_0 + \alpha t,$

We get $\omega = \alpha t$

$\therefore \alpha^2 t^2 = \mu\alpha \quad \Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$



7. (c) The applied force shifts the normal reaction to one corner as shown. At this situation, the cubical block starts topping about O. Taking torque about O



$F \times L = mg \times \frac{L}{2} \Rightarrow \quad F = \frac{mg}{2}$

8. (d) Moment of inertia about the diameter of the circular loop (ring) $= \frac{1}{2}MR^2$

Using parallel axis theorem

The moment of inertia of the loop about XX' axis is

$$I_{XX'} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

Where M = mass of the loop and R = radius of the loop

Here $M = L\rho$ and $R = \frac{L}{2\pi}$;

$$\therefore \quad I_{XX'} = \frac{3}{2}(L\rho)\left(\frac{L}{2\pi}\right)^2 = \frac{3L^3\rho}{8\pi^2}$$

9. (b) The $M.I.$ about the axis of rotation is not constant as the perpendicular distance of the bead with the axis of rotation increases.

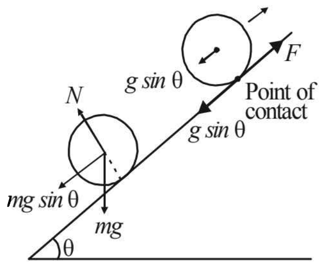
Also since no external torque is acting.

$\therefore \tau_{ext} = \frac{dL}{dt} \Rightarrow L = \text{constant} \Rightarrow I\omega = \text{constant}$

Since, I increases, ω decreases.

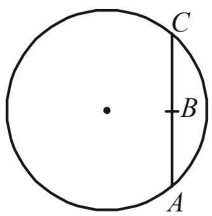
10. (a) The mass distribution of this sector is same about the axis of rotation as that of the complete disc about the axis. Therefore the formula remains the same as that of the disc.

11. (b) Imagine the cylinder to be moving on a frictionless surface. In both the cases the acceleration of the centre of mass of the cylinder is $g \sin\theta$. This is also the acceleration of the point of contact of the cylinder with the inclined surface. Also no torque (about the centre of cylinder) is acting on the cylinder since we assumed the surface to be frictionless and the forces acting on the cylinder is mg and N which pass through the centre of cylinder. Therefore the net movement of the point of contact in both the cases is in the downward direction as shown. Therefore the frictional force will act in the upward direction in both the cases.



Note : In general we find the acceleration of the point of contact due to translational and rotational motion and then find the net acceleration of the point of contact. The frictional force acts in the opposite direction to that of net acceleration of point of contact.

12. (b) Since there is no external torque, angular momentum remains conserved. As moment of inertia initially decreases and then increases, so ω will increase initially and then decreases.



Note : The $M.I.$ of the system decreases when the tortoise move from A to B and then increases from B to A.

So the variation of ω is nonlinear.

13. (a) Change in angular momentum of the system = angular impulse given to the system about the centre of mass

(Angular momentum)$_f$ – (Angular momentum)$_i$

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GP_3481

$= Mv \times \frac{L}{2}$ ... (i)

Let the system starts rotating with the angular velocity ω.

Moment of Inertia of the system about its axis of rotation [centre of mass of the system]

$$= M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{2ML^2}{4} = \frac{ML^2}{2}$$

From (i) $I\omega - 0 = Mv\frac{L}{2}$

$$\Rightarrow \quad \omega = \frac{Mv}{I} \times \frac{L}{2} = \frac{Mv}{ML^2/2} \times \frac{L}{2} = \frac{v}{L}$$

14. **(a)** The net force acting on a particle undergoing uniform circular motion is centripetal force which always passes through the centre of the circle. The torque due to this force about the centre is zero, therefore, $\vec{L}$ is conserved about O.

15. **(b)** **KEY CONCEPT :** $(K.E.)_{rotation} = \frac{L^2}{2I}$.

Here , L = constant

∴ $(K.E.)_{rotational} \times I$ = constant.

When I is doubled, $K.E._{rotational}$ becomes half.

16. **(b)** **Note :** In pure rolling, the point of contact is the instantaneous centre of rotation of all the particles of the disc. On applying $v = r\omega$

We find ω is same for all the particles then $v \propto r$.

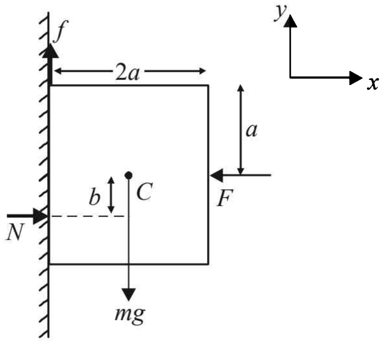
Farther the particles from O, higher is its velocity.

17. **(d)** The cubical block is in equilibrium.

For translational equilibrium

(a) $\Sigma F_x = 0 \Rightarrow F = N$

(b) $\Sigma F_y = 0 \Rightarrow f = mg$



For Rotational Equilibrium

$\Sigma\tau_c = 0$

Where τ_c = torque about c.m.

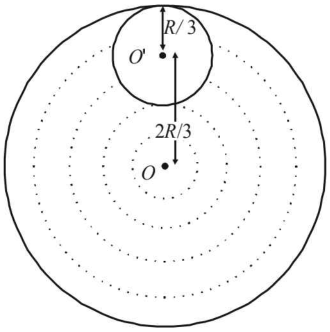
Torque created by frictional force (f) about $C = f \times a$ in clockwise direction.

There should be another torque which should counter this torque. The normal reaction N on the block acts as shown. This will create a torque $N \times b$ in the anticlockwise direction.

Such that $f \times a = N \times b$

Note : The normal force does not act through the centre of the body always. The point of application of normal force depends on all the forces acting on the body.

18. **(a)** Let σ be the mass per unit area.



The total mass of the disc

$= \sigma \times \pi R^2 = 9M$

The mass of the circular disc cut

$$= \sigma \times \pi\left(\frac{R}{3}\right)^2 = \sigma \times \frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it. Moment of inertia (I_1) of the complete disc = $\frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

$M.I.$ of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$$

∴ $M.I.$ (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

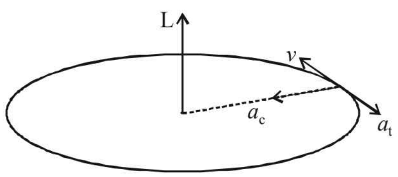
[Using perpendicular axis theorem]

∴ The total $M.I.$ of the system about an axis passing through O and perpendicular to the plane of the disc is $I = I_1 + I_2$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

$$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$$

19. **(b)** Since v is changing (decreasing), L is not conserved in magnitude. Since it is given that a particle is confined to rotate in a circular path, it cannot have spiral path. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.



The direction of $\vec{L}$ remains same even when the speed decreases.

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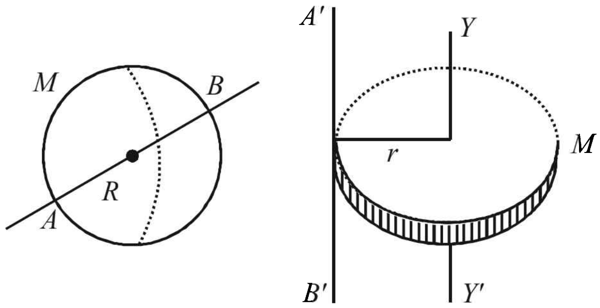
20. (b)



For solid sphere

$$I_{AB} = \frac{2}{5}MR^2 = I \qquad \text{(given)} \qquad \text{... (i)}$$

For solid disc

$$I_{A'B'} = I_{YY'} + Mr^2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$$

$$I_{AB} = I_{A'B'} \qquad \text{(given)} \qquad \text{... (ii)}$$

From (i) and (ii),

$$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2 \Rightarrow r = \frac{2}{\sqrt{15}}R$$

21. (d) By the concept of energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mg\left(\frac{3v^2}{4g}\right)$$

$$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}mv^2 \quad [\because v = R\omega]$$

$$\therefore \quad \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{1}{4}mv^2$$

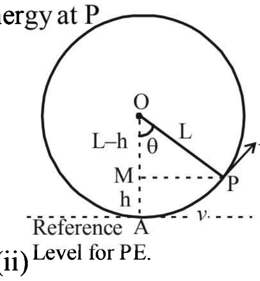
$$\Rightarrow I = \frac{1}{2}mR^2$$

This is the formula of the moment of inertia of the disc.

22. (d) This is the case of vertical motion when the body just completes the circle. Here

$$v = \sqrt{5gL} \qquad \text{..... (i)}$$

Applying energy conservation,
Total energy at A = Total energy at P



$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgh.$$

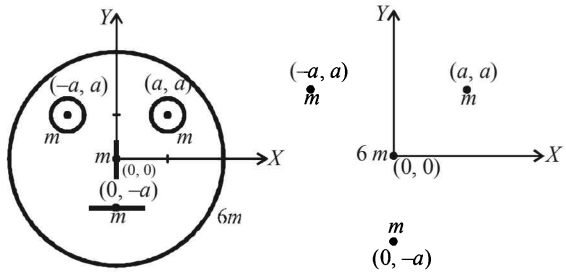
$$\Rightarrow h = \frac{3v^2}{8g}$$

$$= \frac{3}{8g} \times 5gL = \frac{15L}{8} \quad \text{...(ii)}$$

$$\text{In } \Delta\,OPM,\ \cos\theta = \frac{L-h}{L} = \frac{L - \frac{15L}{8}}{L} = \frac{-7}{8}$$

Therefore, the value of θ lies in the range $\frac{3\pi}{4} < \theta < \pi$

23. (a) The system is made up of five bodies (three circles and two straight lines) of uniform mass distribution. Therefore we assume the system to be made up of five point masses where the mass of each body is considered at its geometrical centre.



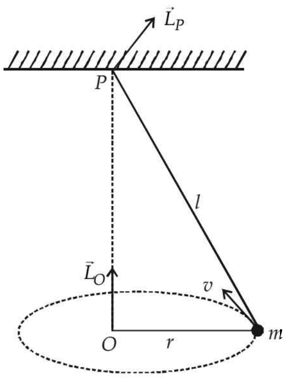
The y-coordinate of the centre of mass is

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4 + m_5y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$\therefore \quad y_{cm} = \frac{6m \times 0 + m \times 0 + m \times a + m \times a + m(-a)}{6m + m + m + m + m}$$

$$= \frac{ma}{10m} = \frac{a}{10}$$

24. (c) The angular momentum of the mass m about O is $mr^2\omega$ and is directed toward $+z$ direction for all locations of m.
The angular momentum of mass m about P is mvl and is directed for the given location of m as shown in the figure.

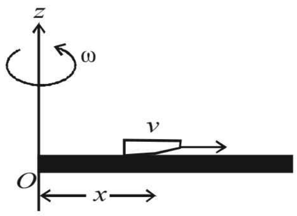


The direction of $\vec{L}_P$ remains changing for different locations of m.

25. (b) We know that $|\vec{\tau}| = \left|\frac{d\vec{L}}{dt}\right|$ where $L = I\omega$

$$\therefore \quad \tau = \frac{d}{dt}(I\omega) = \omega\frac{dI}{dt} \qquad \text{...(i)}$$

From the situation it is clear that the moment of inertia for (rod + insect) system is increasing.



Let at any instant of time 't', the insect is at a distance x from O. At this instant, the moment of inertia of the system is

$$I = \frac{1}{3}ML^2 + mx^2 \qquad \text{...(ii)}$$

From (i) & (ii)

$$\tau = \omega\frac{d}{dt}\left[\frac{1}{3}ML^2 + mx^2\right] = \omega\, m\frac{d}{dt}(x^2)$$

$$= 2\omega mx\frac{dx}{dt} = 2\omega mxv$$

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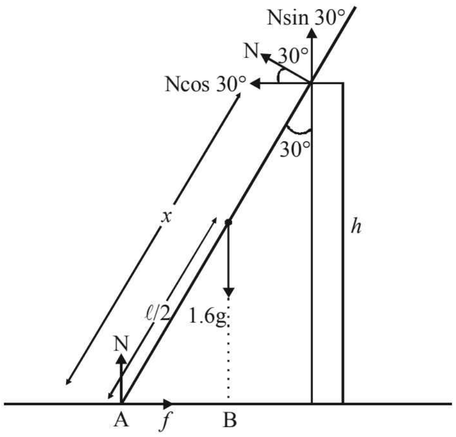
$= 2\omega m v^2 t$ $[\because x = vt]$

$\therefore \quad \tau \propto t$ (till $t = T$)

When the insect stops moving, $\vec{L}$ does not change and therefore τ becomes constant.

26. **(d)** Considering the normal reaction of the floor and wall to be N and with reference to the figure.



By vertical equilibrium.

$$N + N\sin 30° = 1.6\,g \Rightarrow N = \frac{3.2g}{3} \quad \ldots (i)$$

By horizontal equilibrium

$$f = N\cos 30° = \frac{\sqrt{3}}{2}N = \frac{16\sqrt{3}}{3} \text{ From (i)}$$

Taking torque about A we get

$1.6g \times AB = N \times x$

$$1.6\,g \times \frac{\ell}{2}\cos 60° = \frac{3.2g}{3} \times x \quad \therefore \frac{3\ell}{8} = x \quad \ldots (ii)$$

$$\text{But } \cos 30° = \frac{h}{x} \qquad \therefore x = \frac{h}{\cos 30°} \quad \ldots (iii)$$

$$\text{From (ii) and (iii) } \frac{h}{\cos 30°} = \frac{3\ell}{8} \quad \therefore \frac{h}{\ell} = \frac{3\sqrt{3}}{16}$$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(d)** We know that $F_{ext} = Ma_{c.m.}$ ... (i)

We consider the two particles in a system. Mutual force of attraction is a internal force. There are no external forces acting on the system. From (i)

$a_{c.m.} = 0 \Rightarrow v_{c.m.} =$ constant.

Since, initially the $v_{c.m.} = 0$

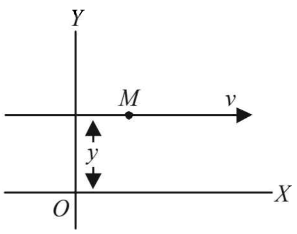
$\therefore$ Finally $v_{c.m.} = 0$

2. **(b)** Angular momentum

$\vec{L} = \vec{r} \times \vec{p}$

$L =$ Momentum $\times$ perpendicular distance of line of action of momentum w.r.t point of rotation

$L = Mv \times y.$



The quantities on the right side of the equation are not changing.

The magnitude is constant. The direction is also constant.

3. **(a, c)** When the cycle is not pedalled but the cycle is in motion (due to previous effort) the wheels move in the direction such that the centre of mass of the wheel move forward. Rolling friction will act in the opposite direction to the relative motion of the centre of mass of the body with respect to ground. Therefore the rolling friction will act in backward direction in both the wheels. The sliding friction will act in the forward direction of rear wheel during pedalling.

4. **(b, d)** Angular momentum = (momentum) $\times$ (perpendicular distance of the line of action of momentum from the axis of rotation)

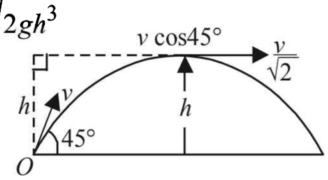
Angular momentum about O

$$L = \frac{mv}{\sqrt{2}} \times h \quad \ldots (i)$$

$$\text{Now, } h = \frac{v^2 \sin^2\theta}{2g} = \frac{v^2}{4g} \quad [\because \theta = 45°] \quad \ldots (ii)$$

From (i) and (ii)

$$L = \frac{m}{\sqrt{2}}(2\sqrt{gh})\,h = m\sqrt{2gh^3}$$



Also, from (i) and (ii)

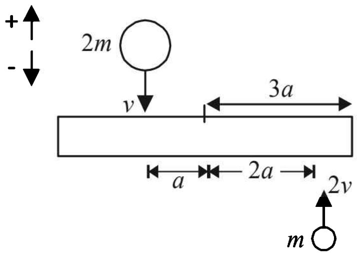
$$L = \frac{mv}{\sqrt{2}} \times \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}\,g}$$

5. **(a, c, d)**

Applying conservation of linear momentum

$2m(-v) + m(2v) + 8m \times 0 = (2m + m + 8m)v_c$

$\Rightarrow v_c = 0$



Applying conservation of angular momentum about centre of mass

$2mv \times a + m(2v) \times 2a = I\omega$ ...(i)

$$\text{Where } I = \frac{1}{12}(8m) \times (6a)^2 + 2m \times a^2 + m \times 4a^2$$

$I = 30ma^2$ ...(ii)

From (i) and (ii)

$$2mv(a) + m(2v) \times 2a = 30ma^2 \times \omega \Rightarrow \omega = \frac{v}{5a}$$

$$\text{Energy after collision, } E = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times 30\,ma^2 \times \frac{v^2}{25a^2} = \frac{3mv^2}{5}$$

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6. **(a, b, c)** To find the moment of inertia of *ABCD* about an axis passing through the centre *O* and perpendicular to the plane of the plate, we use perpendicular axis theorem. If we consider *ABCD* to be in the *X-Y* plane then we know that

$I_{zz'} = I_{xx'} + I_{yy'}$

$\therefore \quad I_{zz'} = I_1 + I_2$

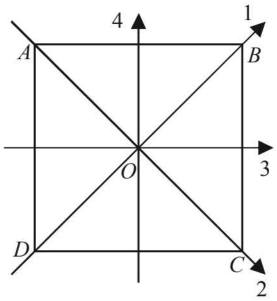
Also, $I_{zz'} = I_3 + I_4$

Adding (i) and (ii),

$2I_{zz'} = I_1 + I_2 + I_3 + I_4$

But $I_1 = I_2$ and $I_3 = I_4$

(By symmetry)

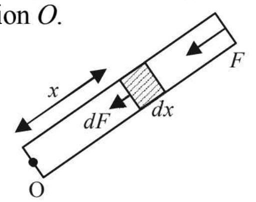


$\therefore \quad 2I_{zz'} = I_1 + I_1 + I_3 + I_3$

$= 2I_1 + 2I_3$

$\Rightarrow \quad I_{zz'} = I_1 + I_3$

7. **(a)** The force acting on the mass of liquid *dm* of length *dx* at a distance *x* from the axis of rotation *O*.



$dF = (dm)\, x\, \omega^2$

$\therefore \quad dF = \dfrac{M}{L} dx \times x\omega^2$

where $\dfrac{M}{L}$ is mass of liquid in unit length.

$\therefore$ The force acting at the other end is for the whole liquid in tube.

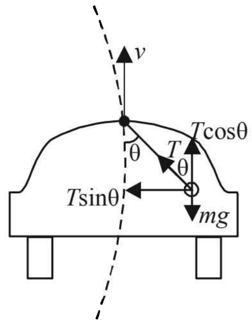
$$F = \int_0^L \frac{M}{L}\omega^2 x\, dx = \frac{M}{L}\omega^2 \int_0^L x\, dx$$

$$= \frac{M}{L}\omega^2 \left[\frac{x^2}{2}\right]_0^L = \frac{M}{L}\omega^2 \left[\frac{L^2}{2} - 0\right] = \frac{ML\omega^2}{2}$$

8. **(c)** When the car is moving in a circular horizontal track of radius 10 m with a constant speed, then the bob is also undergoing a circular motion. The bob is under the influence of two forces.

(i) *T* (tension in the rod)

(ii) *mg* (weight of the bob)



Resolving tension, we get

$T\cos\theta = mg$ ... (i)

And $T\sin\theta = \dfrac{mv^2}{r}$ ... (ii)

(Here $T\sin\theta$ is producing the necessary centripetal force for circular motion)

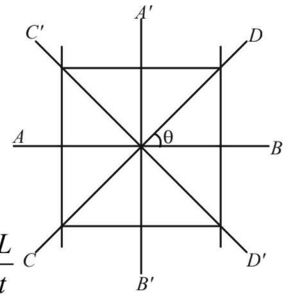
Dividing (ii) by (i), we get

$$\tan\theta = \frac{v^2}{rg} = \frac{10 \times 10}{10 \times 10} = 1 \Rightarrow \theta = 45°$$

9. **(a)** $A'B' \perp AB$ and $C'D' \perp CD$

From symmetry $I_{AB} = I_{A'B'}$ and $I_{CD} = I_{C'D'}$

From theorem of perpendicular axes,



$I_{zz} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$

$\Rightarrow 2I_{AB} = 2I_{CD}$

$\therefore \quad I_{AB} = I_{CD}$

10. **(a, b, c)** **KEY CONCEPT** $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

Given that

$$\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$$

From cross-product rule, $\dfrac{d\vec{L}}{dt}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{L}$.

By the dot product definition

$\vec{L}.\vec{L} = L^2$

Differentiating with respect to time

$$\vec{L}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt} \Rightarrow 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

Since, $\dfrac{d\vec{L}}{dt}$ i.e. $\vec{\tau}$ is perpendicular to $\vec{L}$

$\therefore \quad \vec{L}\cdot\dfrac{d\vec{L}}{dt} = 0 \Rightarrow \dfrac{dL}{dt} = 0$

$\Rightarrow \quad L =$ constant

Thus, the magnitude of *L* always remains constant.

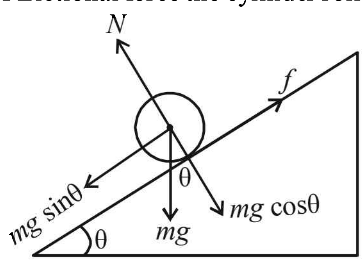
As $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$,

Also, $\vec{L}$ is perpendicular to $\vec{A}$

$\because \vec{L} \perp \vec{A} \quad \therefore \vec{L}.\vec{A} = 0$

Thus, it can be concluded that component of $\vec{L}$ along $\vec{A}$ is zero i.e., always constant.

11. **(c, d)** As shown in the figure, the component of weight $mg\sin\theta$ tends to slide the point of contact (of the cylinder with inclined plane) along its direction. The sliding friction acts in the opposite direction to oppose this relative motion. Because of frictional force the cylinder rolls.



Thus frictional force adds rotation but hinders translational motion.

Applying $F_{net} = ma$ along the direction of inclined plane, we get $mg\sin\theta - f = ma_c$,

where a_c = acceleration of centre of mass of the cylinder

$\therefore \quad f = mg\sin\theta - ma_c$ ...(i)

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But $a_c = \dfrac{g\sin\theta}{1+\dfrac{I_c}{mR^2}} = \dfrac{g\sin\theta}{1+\dfrac{mR^2/2}{mR^2}} = \dfrac{2}{3}g\sin\theta$ ...(ii)

From (i) and (ii), $f = \dfrac{mg\sin\theta}{3}$

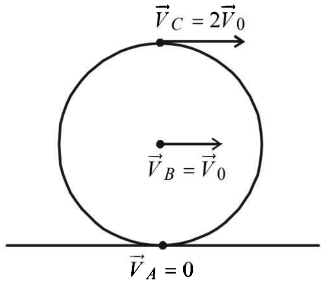
If θ is reduced, frictional force is reduced.

12. **(a)** $\sum \vec{F}_{ext} = \dfrac{d\vec{p}_{system}}{dt}$

Given $\sum \vec{F}_{ext} = 0 \Rightarrow \vec{p}_{system} = \text{Constant}$

Due to internal forces acting in the system, the kinetic and potential energy may change with time.
Also zero external force may create a torque if the line of action of forces are along different direction. Thus the torque will change the angular momentum of the system.

13. **(b, c)**



If $\vec{V}_0$ is the velocity of centre of the sphere, then

$\vec{V}_C = 2\vec{V}_0$, $\vec{V}_B = \vec{V}_0$ and $\vec{V}_A = 0$

$\therefore \quad \vec{V}_C - \vec{V}_B = 2\vec{V}_0 - \vec{V}_0 = \vec{V}_0$

$\vec{V}_B - \vec{V}_A = \vec{V}_0 - \vec{0} = \vec{V}_0$

$\therefore \quad \vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

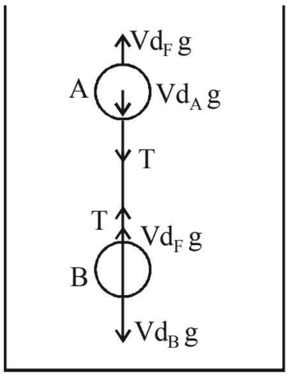
(b) is the correct option.

Now, $|\vec{V}_C - \vec{V}_A| = |2\vec{V}_0 - 0| = |2\vec{V}_0| = 2|\vec{V}_0|$

and $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(c) is the correct option.

14. **(a,b,d)**



Let V be the volume of spheres.

For equilibrium of A :

$T + Vd_A g = Vd_f g$

$\therefore \quad T = V_g(d_f - d_A)$ ...(1)

For $T > 0$, $d_f > d_A$ or $d_A < d_f$

(a) is the correct option

For equilibrium of B :

$T + Vd_f g = Vd_B g$

$\therefore T = Vg(d_B - d_f)$ ...(2)

For $T > 0$, $d_B > d_f$

(b) is the correct option

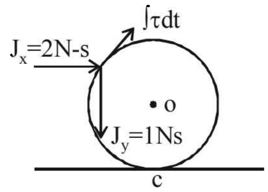
From (1) & (2) $Vg(d_f - d_A) = Vg(d_B - d_f)$

$\therefore \quad d_f - d_A = d_B - d_f$

$\therefore 2d_f = d_A + d_B$

(d) is the correct option.

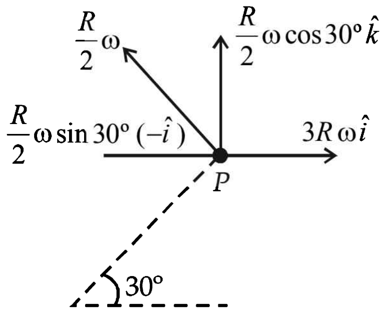
15. **(c)** The frictional force between the ring and the ball is impulsive. The angular impulse created by this force tends to decrease the angular speed of the ring about O.



After the collision the angular speed decreases but the ring remains rotating in the anticlock wise direction. Therefore the friction between the ring and the ground (at the point of contact) is towards left.

16. **(a, b)** For rolling motion, the velocity of the point of contact with respect to the surface should be zero. For this

$3R\omega(-\hat{i}) + \vec{v}_0 = 0 \qquad \therefore \quad \vec{v}_0 = 3R\omega\,\hat{i}$



A shown in the figure, the point P will have two velocities

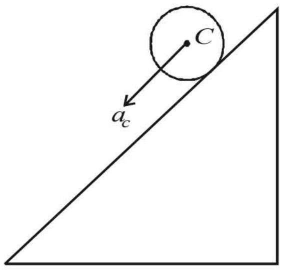
(i) $3R\omega\,\hat{i}$ (due to translational motion)

(ii) $\dfrac{R}{2}\omega$ making an angle of 30° with the vertical due to rotation

$\therefore \quad \vec{v_P} = \left[3R_\omega\,\hat{i} - \dfrac{R_\omega}{4}\hat{i}\right] + \dfrac{\sqrt{3}R_\omega}{4}\hat{k}$

$= \dfrac{11}{4}R_\omega\,\hat{i} + \dfrac{\sqrt{3}}{4}R_\omega\,\hat{k}$

17. **(d)** The acceleration of the center of mass of cylinder rolling down an inclined plane is

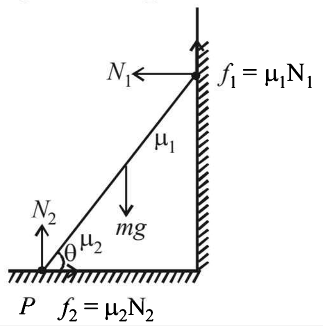


$a_c = \dfrac{g\sin\theta}{1+\dfrac{I_P}{MR^2}}$

Here $I_P > I_Q$ because in case of P the mass is concentrated away from the axis.

$\therefore \quad a_P < a_Q \Rightarrow v_P < v_Q \Rightarrow \omega_P < \omega_Q$

18. **(c, d) When $\mu_1 \neq 0$ and $\mu_2 \neq 0$**



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$N_1 = \mu_2 N_2$ [∵ horizontal equilibrium]

$mg = N_2 + \mu_1 N_1$ [∵ vertical equilibrium]

Solving the above equation we get

$$N_2 = \frac{mg}{1+\mu_1\mu_2}$$

∴ (c) is the correct option.

When $\mu_1 = 0$

Taking torque about P we get

$$mg \times \frac{l}{2}\cos\theta = N_1 \times l \sin\theta$$

$$N_1 \tan\theta = \frac{mg}{2}$$

∴ (d) is correct

19. (d) Applying conservation of angular mumentum about the axis

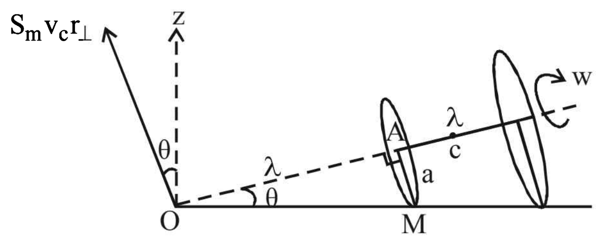
$$MR^2 \times \omega = MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8} r^2 \times \frac{8\omega}{9}$$

$$\Rightarrow \quad r = \frac{4R}{5}$$

D is the correct option

20. (a, c) In ΔOAM, $OM = \sqrt{l^2 + a^2} = \sqrt{2ha^2 + a^2} = 5a$

The circumference of a circle of radius OM will be $2\pi(5a) = 10\pi a$.



For completing this circle once, the smaller disc will have to take $\frac{10\pi a}{2\pi a} = 5$ rounds.

Therefore the C.M. of the assembly rotates about z-axis with an angular speed of w/5.

The angular momentum about the C.M. of the system

$$L_c = I_c w = \left[\frac{1}{2}ma^2\right]\omega$$

$$+\left[\frac{1}{2} \times 4m \times (2a)^2\right] \times \omega = \frac{17ma^2\omega}{2}$$

Now $v_c = \frac{m \times \omega a + 4m \times 2\omega a}{5m} = \frac{9\omega a}{5}$

and $r_\perp = \frac{ml + 4m \times 2l}{5m} = \frac{9l}{5}$

$$L \text{ of C.M} = \frac{5m \times 9\omega a}{5} \times \frac{9l}{5} = 81m\omega a^2 \times \frac{\sqrt{24}}{5}$$

$$L_Z = \frac{81m\omega a^2\sqrt{24}}{5}\cos\theta - I_c\omega\sin\theta$$

$$= 81m\omega a^2\sqrt{\frac{24}{5}} \times \sqrt{\frac{24}{5}} - \frac{17ma^2\omega}{10} = \frac{1134}{50}m\omega a^2$$

21. (a, b)

$$\vec{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$$

$$\vec{r} = \frac{10}{3}t^3\hat{i} + 5t^2\hat{j}m$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 10t^2\hat{i} + 10t\hat{j}\ ms^{-1}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 20t\hat{i} + 10\hat{j}ms^{-2}$$

At t = 1s

$$\vec{r}_{t=1} = \frac{10}{3}\hat{i} + 5\hat{j}m\ ;$$

$$\vec{v}_{t=1} = 10\hat{i} + 10\hat{j}\ ms^{-1}$$

$$\vec{p}_{t=1} = \hat{i} + \hat{j}\ \ kgms^{-1}$$

$$\vec{a}_{t=1} = 20\hat{i} + 10\hat{j}\ ms^{-2}$$

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{k}\left[\frac{10}{3} - 5\right] = -\frac{5}{3}\hat{k}\ \text{kgms}^{-1}$$

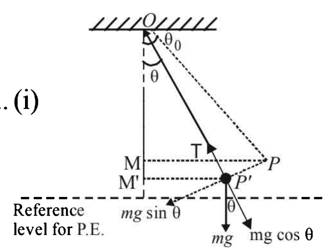
$$\vec{F} = m\vec{a} = (2\hat{i} + \hat{j})N$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 2 & 1 & 0 \end{vmatrix} = \hat{k}\left[\frac{10}{3} - 10\right] = \frac{-20}{3}\hat{k}\ \text{N–m}$$

E. Subjective Problems

1. $T - mg\cos\theta = \frac{mv^2}{\ell}$

∴ $T = \frac{mv^2}{\ell} + mg\cos\theta$ ... (i)



In ΔOPM, $\cos\theta_0 = \frac{OM}{\ell}$

$\Rightarrow OM = \ell\cos\theta_0$

In ΔOP'M', $\cos\theta = \frac{OM'}{\ell}$

$\Rightarrow OM' = \ell\cos\theta$

$OM' - OM = \ell(\cos\theta - \cos\theta_0)$

Loss in potential energy = Gain in kinetic energy (Activity P to P')

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$\Rightarrow \quad mg\ell(\cos\theta - \cos\theta_0) = \frac{1}{2}mv^2$

$\Rightarrow \quad v^2 = 2g\ell(\cos\theta - \cos\theta_0)$ ... (ii)

From (i) and (ii)

$$T = \frac{m}{\ell} \times 2g\,\ell(\cos\theta - \cos\theta_0) + mg\cos\theta$$

$\therefore \quad T = 3mg\cos\theta - 2\,mg\cos\theta_0$

$\Rightarrow \quad T = mg\,(3\cos\theta\ - 2\cos\theta_0\,)$

From equation (i) it is clear that the tension is maximum when $\cos\theta\ = 1$ i.e., $\theta = 0°$

$\therefore\ T = mg$

Hence, $T_{max} = \frac{mv^2}{\ell} + mg$ ... (iii)

From eqn. (ii)

$v^2 = 2g\ell(1 - \cos\theta_0)$ ... (iv)

From (iii) and (iv)

$$T_{max} = \frac{m}{\ell}[2g\ell(1 - \cos\theta_0)] + mg$$

$\therefore \quad T_{max} = 3mg - 2mg\cos\theta_0$

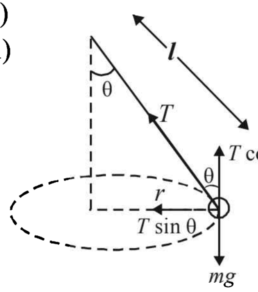
$80\ = 3 \times 40 - 2 \times 40\ \cos\theta_0$

$\Rightarrow \quad 80\ \cos\theta_0 = 40 \Rightarrow \cos\theta_0 = \frac{1}{2} \ \Rightarrow \ \theta_0 = 30°$

2. Suppose mass m moves around a circular path of radius r. Let the string makes an angle θ with the vertical. Resolving tension T, we get

and, $T\sin\theta = mr\omega^2$ ... (i)

$T\cos\theta = mg$ ... (ii)



$\therefore \quad \tan\theta = \frac{r\omega^2}{g}$

From diagram, $\sin\theta = \frac{r}{\ell}$

$\Rightarrow \quad r = \ell\sin\theta$

$\therefore \quad \tan\theta = \ell\sin\theta\frac{\omega^2}{g}$

$\omega^2 = \frac{\tan\theta . g}{\ell\sin\theta} \quad \omega = \sqrt{\frac{g}{\ell\cos\theta}}$

$\Rightarrow \nu = \frac{1}{2\pi}\sqrt{\frac{g}{\ell\cos\theta}}$ ... (iii)

From (ii), $T\cos\theta = mg$.

For M to remain stationary, $T = Mg$

$\therefore \quad Mg\cos\theta = mg$

$\Rightarrow \quad \cos\theta = \frac{m}{M}$ ... (iv)

From (iii) and (iv), $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}\frac{M}{m}}$

3. Let σ be the mass per unit area.

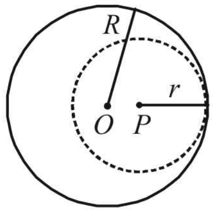
Then the mass of the whole disc $= \sigma \times \pi R^2$

Mass of the portion removed $= \sigma \times \pi r^2$

$R = 28$ cm; $r = 21$ cm; $OP = 7$ cm

Taking O as the origin

The position of c.m.



$$x = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

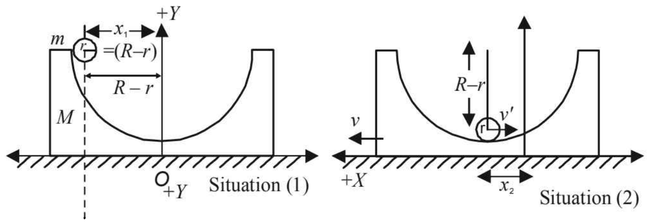
$$= \frac{\sigma \times \pi R^2(0) - \sigma \times \pi r^2 \times 7}{\sigma\pi R^2 - \sigma\pi r^2}$$

$$= \frac{-(21)^2 \times 7}{(28)^2 - (21)^2} = -\frac{21 \times 21 \times 7}{7 \times 49} = -9\text{ cm}$$

This means that the c.m. lies at a distance of 9 cm from the origin towards left.

4. C.M. of the system of two bodies in situation (i) in x-coordinate

$$x_C = \frac{M \times 0 + mx_1}{M + m} = \frac{mx_1}{M + m} \quad \text{... (i)}$$



C.M. of the system in situation (ii) in x-coordinate is

$$x'_C = \frac{M \times x_2 + m \times x_2}{M + m} = x_2 \quad \text{... (ii)}$$

Since no external force is in x-direction

$\therefore \quad x_C = x'_C$

$\therefore \quad x_2 = \frac{mx_1}{M + m} = \frac{m(R - r)}{M + m}$

Applying conservation of linear momentum,

Initial Momentum = Final Momentum

$0 = MV - mv$

$\therefore \quad v = \frac{MV}{m}$ .... (iii)

Applying the concept of conservation of energy, we get

Loss in P.E. of mass m = Gain in K.E. of mass M and Gain in K.E. of mass m

$\Rightarrow \quad mg\,(R - r) = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$

$\Rightarrow \quad 2mg\,(R - r) = MV^2 + m\frac{M^2V^2}{m^2}$ [from (iii)]

$\Rightarrow \quad 2mg\,(R - r) = MV^2 + \frac{M^2V^2}{m}$

$2mg\,(R - r) = MV^2\left[1 + \frac{M}{m}\right] = MV^2\left[\frac{m + M}{m}\right]$

$\Rightarrow \quad \frac{2m^2g\,(R - r)}{M\,(m + M)} = V^2 \ \Rightarrow \quad V = m\sqrt{\frac{2g\,(R - r)}{M\,(m + M)}}$

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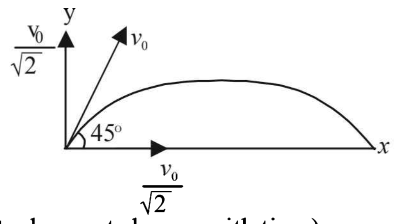
5. The angular momentum is given by $L = xp_y - yp_x$
$= m[xv_y - yv_x]$

(x, y) are the coordinates of the particle after time $t = \frac{v_0}{g}$ and v_x, v_y are the components of velocities at that time.

For v_x and v_y



$v_x = v_0 \cos 45° = \frac{v_0}{\sqrt{2}}$

(The horizontal velocity does not change with time)
Applying $v = u + at$ in the vertical direction to find v_y

$$v_y = (v_0 \sin 45°) - g\left(\frac{v_0}{g}\right) = \frac{v_0}{\sqrt{2}} - g \times \frac{v_0}{g} = \frac{v_0}{\sqrt{2}} - v_0$$

For x and y

In horizontal direction $x = v_x \times t$

$$\therefore \quad x = \frac{v_0}{\sqrt{2}} \times \frac{v_0}{g} = \frac{v_0^2}{\sqrt{2}g}$$

In vertical direction applying $S = ut + \frac{1}{2}at^2$

$$y = \frac{v_0}{\sqrt{2}} \times \frac{v_0}{g} - \frac{1}{2}g\frac{v_0^2}{g^2} = \frac{v_0^2}{\sqrt{2}g} - \frac{v_0^2}{2g}$$

Putting the values in the above equation

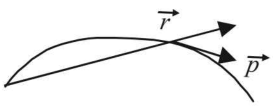
$$L = m\left[\frac{v_0^2}{\sqrt{2}g} \times \left(\frac{v_0}{\sqrt{2}} - v_0\right) - \left(\frac{v_0^2}{\sqrt{2}g} - \frac{v_0^2}{2g}\right)\frac{v_0}{\sqrt{2}}\right]$$

$$L = m\left[\frac{v_0^3}{2g} - \frac{v_0^3}{\sqrt{2}g} - \frac{v_0^3}{2g} + \frac{v_0^3}{2\sqrt{2}g}\right]$$

$$L = \frac{mv_0^3}{g}\left[\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right] \qquad L = \frac{-mv_0^3}{2\sqrt{2}g}$$

Now, $\vec{L} = \vec{r} \times \vec{p}$

Note : The direction of L is perpendicular to the plane of motion and is directed away from the reader.



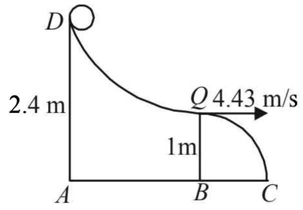
6. **KEY CONCEPT :** Applying law of conservation of energy at point D and point A
$P.E.$ at $D = P.E.$ at $Q + (K.E.)_T + (K.E.)_R$ where
$(K.E.)_T$ = Translational $K.E.$ and $(K.E.)_R$ = Rotational $K.E.$

$$\Rightarrow \quad mg(2.4) = mg(1) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad \text{...(i)}$$

Since the case is of rolling without slipping

$\therefore \quad v = r\omega$

$\therefore \quad \omega = \frac{v}{r}$ where r is the radius of the sphere



Also, $\quad I = \frac{2}{5}mr^2$

Putting in equation (i)

$$mg(2.4-1) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$$

or, $\quad g \times 1.4 = \frac{7v^2}{10} \Rightarrow v = 4.43$ m/s

After point Q, the body takes a parabolic path.
The vertical motion parameters of parabolic motion will be
$u_y = 0 \qquad S_y = 1$m
$a_y = 9.8$ m/s² $\qquad t_y = ?$

$$\therefore \quad S = ut + \frac{1}{2}at^2 \Rightarrow 1 = 4.9\,t_y^2$$

$$t_y = \frac{1}{\sqrt{4.9}} = 0.45 \text{ sec}$$

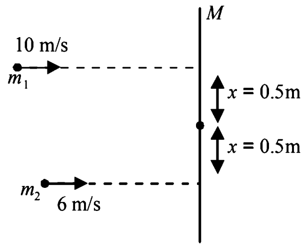
Applying this time in horizontal motion of parabolic path,
$BC = 4.43 \times 0.45 = 2$m

Note : During its flight as a projectile, the sphere continues to rotate because of conservation of angular momentum.

7. Initial Kinetic Energy

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{2}0.08 \times 10^2 + \frac{1}{2}0.08 \times 6^2 + 0 = 5.44 \text{ J} \qquad \text{...(i)}$$



Applying law of conservation of linear momentum during collision
$m_1 \times v_1 + m_2 \times v_2 = (M + m_1 + m_2)\,V_c$
where V_c is the velocity of centre of mass of the bar and particles sticked on it after collision
$0.08 \times 10 + 0.08 \times 6 = (0.16 + 0.08 + 0.08)\,V_c$
$\Rightarrow \quad V_c = 4$ m/s
$\therefore \quad$ Translational kinetic energy after collision

$$= \frac{1}{2}(M + m_1 + m_2)V_c^2 = 2.56 \text{ J} \qquad \text{...(ii)}$$

Applying conservation of angular momentum of the bar and two particle system about the centre of the bar.

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Since external torque is zero, the initial angular momentum is equal to final angular momentum.

Initial angular momentum

$= m_1 v_1 \times x - m_2 v_2 x$

$= 0.08 \times 10 \times 0.5 - 0.08 \times 6 \times 0.5$

$= 0.4 - 0.24 = 0.16 \text{ kg m}^2\text{s}^{-1}$ (In clockwise direction)

Final angular momentum $= I\omega$

$$= \left[\frac{M\ell^2}{12} + m_1 x^2 + m_2 x^2\right]\omega$$

$$= \left[\frac{(0.16)(\sqrt{3})^2}{12} + 0.08 \times (0.5)^2 + (0.08)(0.5)^2\right]\omega$$

$= 0.08\,\omega$

$\therefore \quad 0.08\,\omega = 0.16 \Rightarrow \omega = 2$ rad/s ... (iii)

The rotational kinetic energy

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.08 \times 2^2 = 0.16 \text{ J} \quad \text{... (iv)}$$

The final kinetic energy

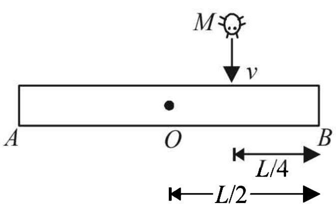
= Translational $K.E.$ + Rotational $K.E.$

$= 2.56 + 0.16 = 2.72$ J

The change in $K.E.$ = Initial $K.E.$ – Final $K.E.$

$= 5.44 - 2.72 = 2.72$ J

8. **(a)** Let us consider the system of homogeneous rod and insect and apply conservation of angular momentum during collision about the point O.



Angular momentum of the system before collision = angular momentum of the system after collision.

$$Mv \times \frac{L}{4} = I\omega$$

Where I is the moment of inertia of the system just after collision and ω is the angular velocity just after collision.

$$\Rightarrow \quad Mv\frac{L}{4} = \left[M\left(\frac{L}{4}\right)^2 + \frac{1}{12}ML^2\right]\omega$$

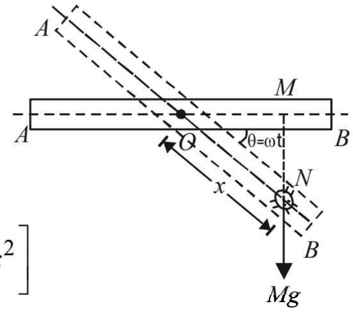
$$\Rightarrow \quad Mv \times \frac{L}{4} = \frac{ML^2}{4}\left[\frac{1}{4} + \frac{1}{3}\right]\omega = \frac{ML^2}{4}\left[\frac{3+4}{12}\right]$$

$$= \frac{ML^2}{4} \times \frac{7}{12} \times \omega \quad \Rightarrow \quad \omega = \frac{12}{7}\frac{v}{L}.$$

(b) **Note :** Initially the torque due to mass OB of the rod (acting in clockwise direction) was balanced by the torque due to mass OA of the rod (acting in anticlockwise direction). But after collision there is an extra mass M of the insect which creates a torque in the clockwise direction, which tends to create angular acceleration in the rod. But the same is compensated by the movement of insect towards B due to which moment of inertia I of the system increases.

Let at any instant of time t the insect be at a distance x from the centre of the rod and the rod has turned through an angle θ $(= \omega t)$ w.r.t its original position.

Instantaneous torque,



$$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega)$$

$$= \omega\frac{dI}{dt}$$

$$= \omega\frac{d}{dt}\left[\frac{1}{12}ML^2 + Mx^2\right]$$

$$= 2M\omega x\frac{dx}{dt} \quad \text{... (i)}$$

This torque is balanced by the torque due to weight of insect.

τ = Force × Perpendicular distance of force with axis of rotation $= Mg \times (OM)$

$= Mg\,(x\cos\theta)$ ... (ii)

From (i) and (ii)

$$2M\omega x\frac{dx}{dt} = Mg\,(x\cos\theta) \Rightarrow dx = \left(\frac{g}{2\omega}\right)\cos\omega t\,dt$$

On integration, taking limits

$$\int_{L/4}^{L/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega} \cos\omega t\,dt$$

when $\quad x = \dfrac{L}{4}, \omega t = 0$

$$[x]_{L/4}^{L/2} = \frac{g}{2\omega^2}[\sin\omega t]_0^{\pi/2\omega}$$

when $\quad x = \dfrac{L}{2}, \omega t = \dfrac{\pi}{2}$

$$\Rightarrow \left(\frac{L}{2} - \frac{L}{4}\right) = \frac{g}{2\omega^2}\left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$\Rightarrow \frac{L}{4} = \frac{g}{2\omega^2} \Rightarrow \omega = \sqrt{\frac{2g}{L}}$$

But $\omega = \dfrac{12}{7}\dfrac{v}{L} \Rightarrow \dfrac{12}{7}\dfrac{v}{L} = \sqrt{\dfrac{2g}{L}} \Rightarrow v = \dfrac{7}{12}\sqrt{2gL}$

$$\Rightarrow v = \frac{7}{12}\sqrt{2 \times 10 \times 1.8} = 3.5 \text{ ms}^{-1}$$

9. **(i)** Initially, the rod stands vertical. A straight disturbance makes the rod to rotate. While rotating, the force acting on the rod are its weight and normal reaction. These forces are vertical forces and cannot create a horizontal motion. Therefore the centre of mass of the rod does not move horizontally. The center of mass moves vertically downwards. Thus the path of the center of mass is a straight line.

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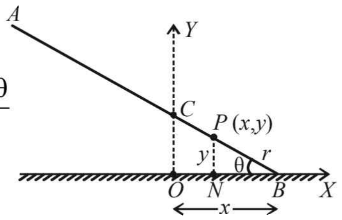
(ii) Trajectory of an arbitrary point of the rod

Consider an arbitrary point P on the rod located at (x, y) and at a distance r from the end B. Let θ be the angle of inclination of the rod with the horizontal at this position.

In ΔBNP, $\sin\theta = \frac{y}{r}$ ... (i)



$$\cos\theta = \frac{x + BN}{L/2} = \frac{x + r\cos\theta}{L/2}$$

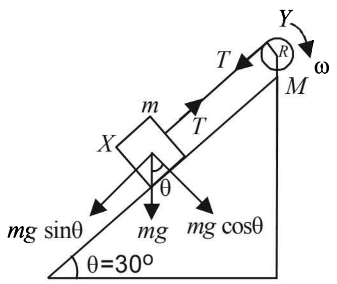
$$\Rightarrow \cos\theta = \frac{x}{\frac{L}{2} - r} \quad \text{... (ii)}$$

From (i) and (ii) $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{y^2}{r^2} + \frac{x^2}{\left(\frac{L}{2} - r\right)^2} = 1$$

This is equation of an ellipse.

10. **(i)** The drum is given an initial velocity such that the block X starts moving up the plane.



As the time passes, the velocity of the block decreases. The linear retardation a, of the block X is given by

$$mg\sin\theta - T = ma \quad \text{... (i)}$$

The linear retardation of the block and the angular acceleration of the drum (α) are related as

$$a = R\alpha \quad \text{... (ii)}$$

where R is the radius of the drum.

The retarding torque of the drum is due to tension T in the string.

$$\tau = T \times R$$

But $\tau = I\alpha$. where $I = M.I.$ of drum about its axis of rotation.

$$\therefore \quad T \times R = \frac{1}{2}MR^2\alpha \quad \text{... (iii)} \quad \left[\because I = \frac{1}{2}MR^2\right]$$

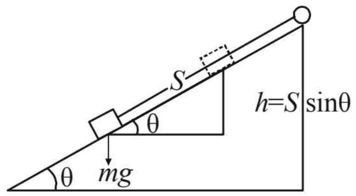
From (ii), $TR = \frac{1}{2}MR^2\frac{a}{R} \Rightarrow a = \frac{2T}{M}$

Substituting this value in (i)

$$mg\sin\theta - T = m \times \frac{2T}{M} \Rightarrow mg\sin\theta = \left(1 + \frac{2m}{M}\right)T$$

$$\therefore \quad T = \frac{(mg\sin\theta) \times M}{M + 2m} = \frac{0.5 \times 9.8 \times \sin 30° \times 2}{2 + 2 \times 0.5} = 1.63\text{ N}$$

(ii) The total kinetic energy of the drum and the block at the instant when the drum is having angular velocity 10 rads^{-1} gets converted into the potential energy of the block.



$$[(K.E.)_{\text{Rotational}}]_{\text{drum}} + \{(K.E.)_{\text{Translational}}\}_{\text{block}} = mgh$$

$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgS\sin\theta$$

$$\frac{1}{2}I\omega^2 + \frac{1}{2}m(R\omega)^2 = mgS\sin\theta \quad [\because v = R\omega]$$

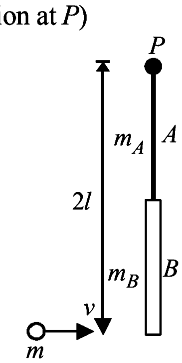
$$\Rightarrow \frac{1}{2}MR^2\omega^2 + \frac{1}{2}mR^2\omega^2 = mgS\sin\theta$$

$$\Rightarrow \frac{1}{2}\frac{R^2\omega^2(M + m)}{mg\sin\theta} = S$$

$$\Rightarrow S = \frac{1}{2} \times \frac{0.2 \times 0.2 \times 10 \times 10(2 + 0.5)}{0.5 \times 9.8 \times \sin 30°} = 1.22\text{ m}$$

11. During collision, the torque of the system about P will be zero because the only force acting on the system is through P (namely weight of rods/mass m/reaction at P)

Given : $\ell = 0.6$ m
$m_A = 0.01$ kg
$m_B = 0.02$ kg
$m = 0.05$ kg



Since, $\tau = \frac{dL}{dt}$ and $\tau = 0$

$\Rightarrow$ L is constant.

Angular momentum before collision $= mv \times 2\ell$ ... (i)

Angular momentum after collision $= I\omega$ ... (ii)

Where I is the moment of inertia of the system after collision about P and ω is the angular velocity of the system.

$M.I.$ about P : $I_1 = M.I.$ of mass m

$I_2 = M.I.$ of rod m_A

$I_3 = M.I.$ of rod m_B

$I = I_1 + I_2 + I_3$

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$$= \left[m(2\ell)^2 + \left\{ m_A\left(\frac{\ell^2}{12}\right) + \left(\frac{\ell}{2}\right)^2 \right\} + \left\{ m_B\left(\frac{\ell^2}{12}\right) + \left(\frac{\ell}{2}+\ell\right)^2 \right\} \right]$$

$$= \left[4m\ell^2 + m_A\left(\frac{\ell^2}{12}+\frac{\ell^2}{4}\right) + m_B\left(\frac{\ell^2}{12}+\frac{9\ell^2}{4}\right) \right]$$

$$= \left[4m\ell^2 + \frac{1}{3}m_A\ell^2 + \frac{7}{3}m_B\ell^2 \right] = 0.09 \text{ kg } m^2$$

From (i) and (ii)

$$I\omega = mv \times 2\ell$$

$$\Rightarrow \quad \omega = \frac{mv \times 2\ell}{I} = \frac{0.05 \times v \times 2 \times 0.6}{0.09} = 0.67\, v$$

Applying conservation of mechanical energy after collision. (Using the concept of mass)

Loss of $K.E.$ = Gain in $P.E.$

$$\frac{1}{2}I\omega^2 = mg(2\ell) + m_A\left(\frac{\ell}{2}\right)g + m_B g\left(\frac{3\ell}{2}\right)$$

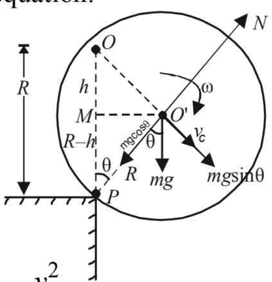
$$\Rightarrow \quad \frac{1}{2} \times 0.09 \times (0.67v)^2$$

$$= \left[0.05 \times 2 + 0.01 \times \frac{1}{2} + 0.02 \times \frac{3}{2}\right] \times 9.8 \times 0.6$$

$$\Rightarrow \quad v = 6.3 \text{ m/s}$$

12. **(a)** Let the original position of centre of mass of the cylinder be O. While rolling down off the edge, let the cylinder be at such a position that its centre of mass is at a position O'. Let $\angle NPO$ be θ. As the cylinder is rolling, the c.m. rotates in a circular path. The centripetal force required for the circular motion is given by the equation.

$$mg\cos\theta - N = \frac{mv_c^2}{R}$$



Where N is the normal reaction and m is mass of cylinder.

The condition for the cylinder leaving the edge is $N = 0$

$$mg\cos\theta = \frac{mv_c^2}{R} \Rightarrow \cos\theta = \frac{v_c^2}{Rg} \qquad \text{... (i)}$$

Applying energy conservation from O to O'.

Loss of potential energy of cylinder

= Gain in translational $K.E.$ + Gain in rotational $K.E.$

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2 \qquad \text{... (ii)}$$

Where I is the moment of inertia of the cylinder about O', its axis of rotation, ω is the angular speed, V_c is the velocity of center of mass.

Also for rolling, $v_c = \omega R$

$$\Rightarrow \omega = \frac{v_c}{R} \qquad \text{... (iii)}$$

$$I = \frac{1}{2}MR^2 \qquad \text{... (iv)}$$

From (ii), (iii) and (iv), we get

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2} \times \frac{1}{2}mR^2 \times \frac{v_c^2}{R^2}$$

$$\Rightarrow \quad gh = \frac{1}{2}v_c^2 + \frac{1}{4}v_c^2 = \frac{3}{4}v_c^2 \quad \Rightarrow v_c^2 = \frac{4gh}{3}$$

In $\Delta O'MP$, $\cos\theta = \frac{R-h}{R}$

$$\Rightarrow h = R(1 - \cos\theta)$$

$$\therefore \quad v_c^2 = \frac{4g}{3}R(1-\cos\theta) \qquad \text{... (v)}$$

From (i) and (v), we get

$$\cos\theta = \frac{4gr}{3Rg}(1-\cos\theta)$$

$$\Rightarrow 3\cos\theta = 4 - 4\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{4}{7}$$

(b) From (v) speed of C.M. of cylinder before leaving contact with edge.

$$v_c^2 = \frac{4gR}{3}\left(1-\frac{4}{7}\right) = \frac{4gR}{7} \quad \Rightarrow v_c = \sqrt{\frac{4gR}{7}}$$

(c) Before the cylinder's c.m. reaches the horizontal line of the edge, it leaves contact with the edge as

$$\theta = \cos^{-1}\frac{4}{7} = 55.15°$$

Therefore the rotational $K.E.$, which the cylinder gains at the time of leaving contact with the edge remains the same in its further motion. Thereafter the cylinder gains translational $K.E.$

Again applying energy conservation from O to the point where c.m. is in horizontal line with edge

$$mgR = \frac{1}{2}I\omega^2 + \frac{1}{2}m(v'_c)^2$$

$$mgR = \frac{1}{2} \times \frac{1}{2}mR^2 \times \left(\sqrt{\frac{4g}{7R}}\right)^2 + \frac{1}{2}m(v'_c)^2$$

$$\because \quad \omega = \frac{v_c}{R} = \sqrt{\frac{4gR/7}{R}}$$

$$\Rightarrow \quad mgR - \frac{mgR}{7} = \text{Translational } K.E. = \frac{6mgR}{7}$$

Also, Rotational $K.E. = \frac{1}{2}I\omega^2 = \frac{mgR}{7}$

$$\therefore \quad \frac{\text{Translational } K.E.}{\text{Rotational } K.E.} = 6$$

13. **KEY CONCEPT :** The concept of center of mass can be applied in this problem.

When small sphere M changes its position to other extreme position, there is no external force in the horizontal direction. Therefore the x-coordinate of c.m. will not change.

$$[x_{c.m.}]_{initial} = [x_{c.m.}]_{final}$$

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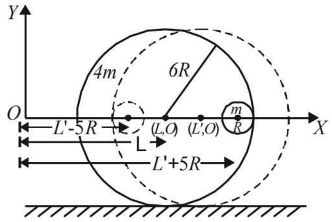


Thin line of sphere represents initial state, dotted line of sphere represents final state.

From (i)

$(x_{c.m.})_{initial} = (x_{c.m.})_{final}$

$\Rightarrow \dfrac{M_1x_1 + M_2x_2}{M_1+M_2} = \dfrac{M_1x'_1 + M_2x'_2}{M_1+M_2}$

$\Rightarrow \dfrac{4m \times L + m\times(5R+L)}{4m+m} = \dfrac{4m\times L' + m\times(L'-5R)}{4m+m}$

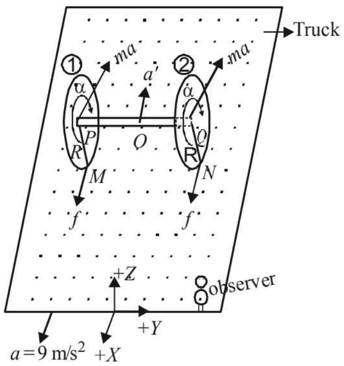
$\Rightarrow 5L + 5R = 5L' - 5R$

$\Rightarrow 5L + 10R = 5L' \quad \Rightarrow L + 2R = L'$

Since, the individual center of mass of the two spheres has a y co-ordinate zero in its initial state and its final state therefore the y-coordinate of c.m. of the two sphere system will remain zero.

Therefore the coordinate of c.m. of bigger sphere is $(L + 2R, 0)$.

14. **(i)** The observer, let us suppose, is on the accelerated frame. Therefore a pseudo force ma is applied individually on each disc on the centre of mass. The frictional force is acting in the $+X$ direction which is producing an angular acceleration α.



The torque acting on the disc is

$\tau = I\alpha = f \times R$

$\Rightarrow f = \dfrac{I\alpha}{R}$ ... (i)

Let a' is the acceleration of c.m. of the disc as seen by the observer. Since the case is of pure rolling and from the perspective of the observer

$a' = \alpha R$ ... (ii)

$\Rightarrow$ From (i) and (ii)

$f = \dfrac{Ia'}{R^2}$ ... (iii)

Applying Newton's law for motion in X-direction

$ma - f = ma'$

$\Rightarrow a' = \left(a - \dfrac{f}{m}\right)$ ... (iv)

Also moment of inertia

$I = \dfrac{1}{2}mR^2$ ... (v)

From (iii), (iv) and (v)

$f = \dfrac{1}{2}\dfrac{mR^2\left(a-\dfrac{f}{m}\right)}{R^2} \Rightarrow 2f = ma - f$

$\Rightarrow 3f = ma \Rightarrow f = \dfrac{ma}{3} = \dfrac{2\times 9}{3} = 6\text{N}$ (In $+X$ direction)

$\vec{f} = (6\hat{i})\text{N}$

(ii) The position vector of point M, taking O as the origin

$\vec{r_m} = -0.1\hat{j} - 0.1\hat{k}$ and position vector of point N

$\vec{r_N} = 0.1\hat{j} - 0.1\hat{k}$

The torque due to friction on disc 1 about O

$\vec{\tau_1} = \vec{r_M} \times \vec{f} = (-0.1\hat{j} - 0.1\hat{k}) \times (6\hat{i})$

$= 0.6(\hat{k} - \hat{j})\, N - m$

The torque due to friction on disc 2 about O

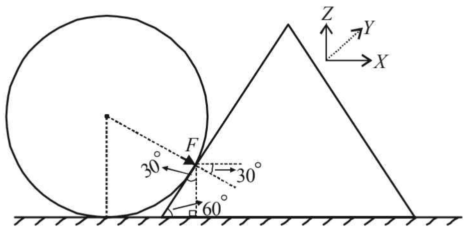
$\vec{\tau_2} = \vec{r_N} \times \vec{f} = (+0.1\hat{j} - 0.1\hat{k}) \times (6\hat{i})$

$= 0.6(-\hat{j} - \hat{k})\, N - m$

The magnitude of torque on each disc

$|\tau_1| = |\tau_2| = 0.6\sqrt{2}\, N - m$

15. **(a)**



Resolving the force F acting on the wedge

$F_x = F\cos 30°;\ F_y = F\sin 30°$

Note : The collision is elastic and since the sphere is fixed, the wedge will return back with the same velocity (in magnitude).

The force responsible to change the velocity of the wedge in X-direction is F_x.

$F_x \times \Delta t = mv - (-mv)$

(Impulse) = (Change in momentum)

$\therefore \quad F_x = \dfrac{2mv}{\Delta t} \Rightarrow F\cos 30° = \dfrac{2mv}{\Delta t} \Rightarrow F = \dfrac{4mv}{\sqrt{3}\,\Delta t}$

In vector terms

$\vec{F} = F_x\hat{i} + F_y(-\hat{k}) = F\cos 30°\hat{i} + F\sin 30°(-\hat{k})$

$= F \times \dfrac{\sqrt{3}}{2}\hat{i} + F \times \dfrac{1}{2}(-\hat{k})$

$\Rightarrow \vec{F} = \dfrac{F}{2}(\sqrt{3}\hat{i} - \hat{k}) = \dfrac{2mv}{\sqrt{3}\Delta t}(\sqrt{3}\hat{i} - \hat{k})$

Taking equilibrium of force in Z-direction (acting on wedge) we get

$F_y + mg = N$

$\Rightarrow N = \dfrac{F}{2} + mg = \dfrac{2mv}{\sqrt{3}\Delta t} + mg$

$N = \left(\dfrac{2mv}{\sqrt{3}\Delta t} + mg\right)\hat{k}$

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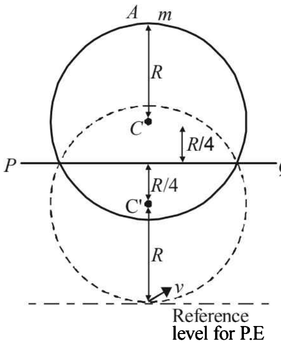
(b) Taking torques on wedge about the c.m. of the wedge.
$F \times h$ – Torque due to $N + mg \times 0 = 0$

$\Rightarrow$ Torque due to $N = F \times h = \frac{4mv}{\sqrt{3}\Delta t} \times h$

16. **KEY CONCEPT :** During the fall, the disc-mass system gains rotational kinetic energy. This is at the expense of potential energy.



Applying energy conservation
Total energy initially = total energy finally

$$mg\left(2R+\frac{2R}{4}\right)+mg\left(R+\frac{2R}{4}\right)=mgR+\frac{1}{2}I\omega^2$$

Where $I = M.I.$ of disc-mass system about PQ

$$mg\times\frac{10R}{4}+mg\frac{6R}{4}=mgR+\frac{1}{2}I\omega^2 \Rightarrow 3mgR=\frac{1}{2}I\omega^2$$

$$\Rightarrow \omega=\sqrt{\frac{6mgR}{I}} \qquad \text{... (i)}$$

$(I)_{PQ}=(I_{disc})_{PQ}+(I_{mass})_{PQ}$

$$=\left[\frac{mR^2}{4}+M\left(\frac{R}{4}\right)^2\right]+m\left(\frac{5R}{4}\right)^2$$

$[\because$ $M.I.$ of disc about diameter $=\frac{1}{4}MR^2]$

$$=\frac{mR^2[4+1+25]}{16}=\frac{15mR^2}{8} \qquad \text{... (ii)}$$

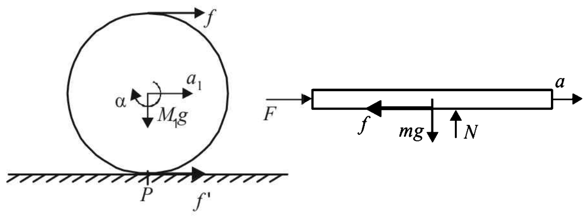
From (i) and (ii)

$$\omega=\sqrt{\frac{6mgR\times 8}{15mR^2}}=\sqrt{\frac{16g}{5R}}$$

Let v be the velocity of mass m at the lowest point of rotation

$$v=\omega\left(R+\frac{R}{4}\right) \quad \therefore \quad v=\sqrt{\frac{16g}{5R}}\times\frac{5R}{4}=\sqrt{5gR}$$

17. The man applies a force F in the horizontal direction on the plank as shown. Therefore the point of contact of the plank with the cylinder will try to move towards right. Therefore the friction force F will act towards left on the plank. To each and every action there is equal and opposite reaction. Therefore a frictional force f will act on the top of the cylinder towards right.



Direction of f' : A force f is acting on the cylinder. This force is trying to move the point of contact P towards right by an acceleration

$$a_{cm}=\frac{f}{M_1} \text{ acting towards right.}$$

At the same time, the force f is trying to rotate the cylinder about its centre of mass.

$f \times R = I \times \alpha$

$$\Rightarrow \alpha=\frac{f\times R}{I}=\frac{f\times R}{\frac{1}{2}M_1R^2}=\frac{2f}{M_1R} \text{ in clockwise direction.}$$

$$\therefore \alpha_{cm}+\alpha R=\frac{f}{M_1}-\frac{2f}{M_1R}\times R=-\frac{f}{M_1}, \text{ i.e., towards left.}$$

Therefore, the point of contact of the cylinder with the ground move towards left. Hence friction force acts towards right on the cylinder.

Note : You can assume any direction of friction at the point of contact and solve the problem. If the value of friction comes out to be positive, our assumed direction is correct otherwise the direction of friction is opposite. The above activity is done so that if only the direction of friction is asked, an approach may be developed.

Applying Newton's law on plank, we get

$F - f = m_2a_2$ ... (i)

Also, $a_2 = 2a_1$ ... (ii)

Because a_2 is the acceleration of topmost point of cylinder and there is no slipping.

Applying Newton's law on cylinder

$M_1a_1 = f + f'$ ... (iii)

The torque equation for the cylinder is

$$f\times R-f'\times R=I\alpha=\frac{1}{2}M_1R^2\times\left(\frac{a_1}{R}\right)$$

$[\because I=\frac{1}{2}M_1R^2$ and $R\alpha=a_1]$

$$\therefore (f-f')R=\frac{1}{2}M_1Ra_1 \Rightarrow f+f'=\frac{1}{2}M_1a_1 \qquad \text{... (4)}$$

Solving equation (iii) and (iv), we get

$$f=\frac{3}{4}M_1a_1 \qquad \text{... (5)}$$

$$\text{and } f'=\frac{1}{4}M_1a_1 \qquad \text{... (6)}$$

From (i) and (iii)

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$$F - f = 2m_2a_1 \Rightarrow F - \frac{3}{4}M_1a_1 = 2m_2a_1$$

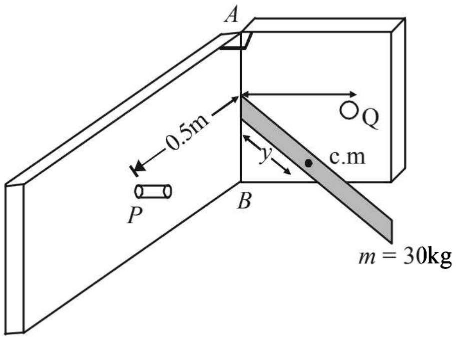
$\therefore\ a_1 = \frac{4F}{3M_1 + 8m_2}$ $\therefore\ a_2 = \frac{8F}{3M_1 + 8m_2}$

From (v) and (vi)

$$f = \frac{3}{4}M_1 \times \frac{4F}{3M_1 + 8m_2} = \frac{3FM_1}{3M_1 + 8m_2}$$

And $f' = \frac{1}{4}M_1 \times a_1 = \frac{FM_1}{3M_1 + 8m_2}$

18. $I_c = 1.2$ kg-m^2



Let y be the distance of c.m. from line AB.
Applying parallel axis theorem of M.I. we get
M.I. of laminar sheet about AB

$I_{AB} = I_{c.m.} + my^2$

$I_{AB} = 12 + 30y^2$ ... (i)

The angular velocity of the laminar sheet will change after every impact because of impulse.
Impulse = Change in linear momentum

$6 = 30(V_f - V_i)$

$6 = 30 \times y(\omega_f - \omega_i)$ ... (ii)

Also, change in angular momentum = Moment of Impulse

$\therefore\ I_{AB}\omega_f - I_{AB}\omega_i =$ Impulse × distance

$I_{AB}(\omega_f - \omega_i) = 6 \times 0.5 = 3$

$$\therefore\ \omega_f = \frac{3}{I_{AB}} + \omega_i = \frac{3}{1.2 + 30y^2} + (-1) \quad \text{... (iii)}$$

Note : Minus sign with ω_i because the direction of laminar plate towards the obstacle is taken as – ve (assumption).
From (ii) and (iii)

$$6 = 30 \times y\left[\frac{3}{1.2 + 30y^2} - 1 + 1\right]$$

$$1 = 5y\left[\frac{3}{1.2 + 30y^2}\right]$$

$\therefore\ 1.2 + 30y^2 = 5y[+3] = 15y$

$\therefore\ 30y^2 - 15y - 1.2 = 0$

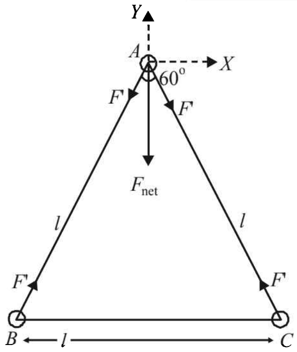
On solving, we get $y = 0.1$ or 0.4

$\therefore\ \omega_f = 1$ rad/s if we put $y = 0.1$ in eq. (ii)

And $\omega_f = 0.5$ rad/s if we put $y = 0.4$ in eq. (ii)
(Not valid as per sign convention)

Now, since the lamina sheet comes back with same angular speed as that of incident angular speed, the sheet will swing in between P and Q infinitely.

19. **(a)** The mass B is moving in a circular path centred at A. The centripetal force $(m\ell\omega^2)$ required for this circular motion is provided by F'. Therefore a force F' acts on A (the hinge) which is equal to $m\ell\omega^2$. The same is the case for mass C. Therefore the net force on the hinge is



$$F_{net} = \sqrt{F'^2 + F'^2 + 2F'F'\cos 60°}$$

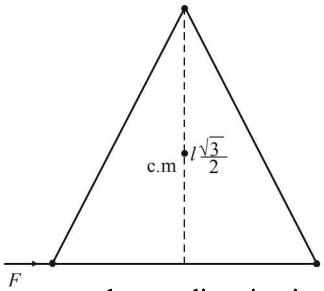
$$F_{net} = \sqrt{2F'^2 + 2F'^2 \times \frac{1}{2}} = \sqrt{3}F' = \sqrt{3}\,m\ell\omega^2$$

(b) The force F acting on B will provide a torque to the system. This torque is

$$F \times \frac{\ell\sqrt{3}}{2} = I\alpha$$

$$F \times \frac{\sqrt{3}\,\ell}{2} = (2m\ell^2)\alpha$$

$$\Rightarrow\ \alpha = \frac{\sqrt{3}}{4} \times \left(\frac{F}{m\ell}\right)$$



The total force acting on the system along x-direction is

$F + (F_{net})_x$

This force is responsible for giving an acceleration a_x to the system.

Therefore,

$$F + (F_{net})_x = 3m(a_x)_{c.m.}$$

$$= 3m\frac{F}{4m} \quad \left(\because\ \alpha_x = \alpha r = \frac{\sqrt{3}}{4}\frac{F}{m\ell} \times \frac{\ell}{\sqrt{3}} = \frac{F}{4}\right)$$

$$= \frac{3F}{4} \quad \therefore\ (F_{net})_x = -\frac{F}{4}$$

$(F_{net})_y$ remains the same as before $= \sqrt{3}\,m\ell\omega^2$.

20. We know that $\vec{\tau} = \frac{d\vec{L}}{dt}$

$\Rightarrow\ \vec{\tau} \times dt = d\vec{L}$

When angular impulse $(\vec{\tau} \times d\vec{t})$ is zero, the angular momentum is constant. In this case for the wooden log-bullet system, the angular impulse about O is constant. Therefore,

[angular momentum of the system]$_{initial}$
= [angular momentum of the system]$_{final}$

$\Rightarrow\ mv \times L = I_0 \times \omega$ ... (i)

where I_0 is the moment of inertia of the wooden log-bullet system after collision about O

$I_0 = I_{\text{wooden log}} + I_{\text{bullet}}$

$= \frac{1}{3}ML^2 + ML^2$ ... (ii)

From (i) and (ii)

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$$\omega = \frac{mv \times L}{\left[\frac{1}{3}ML^2 + mL^2\right]}$$

$$\Rightarrow \omega = \frac{mv}{\left[\frac{ML}{3} + mL\right]} = \frac{3mv}{(M + 3m)L}$$

21. Applying $F_{net} = ma$ in X-direction

$mg \sin\theta - f = ma$ ... (i)

The torque about O will be

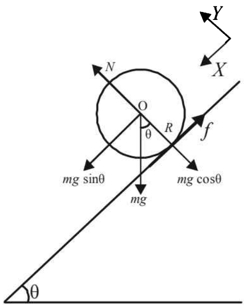
$\tau = f \times R$

$= I\alpha$ ... (ii)

As the case is of rolling

$\therefore \quad a = \alpha R$

$\Rightarrow \alpha = \frac{a}{R}$ ... (iii)



From (ii) and (iii), $f = \frac{Ia}{R^2}$

Substituting this value in (i), we get

$$mg \sin\theta - \frac{Ia}{R^2} = ma$$

$$\Rightarrow a = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{mg\sin\theta}{m + \frac{1}{2}\frac{mR^2}{R^2}} = \frac{2}{3}g\sin\theta$$

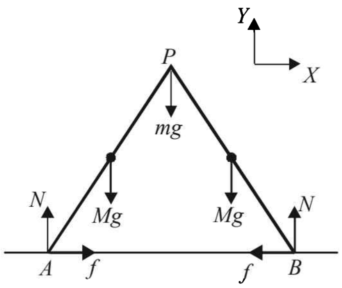
$$\left[\because I = \frac{1}{2}mR^2 \text{ for solid cylinder}\right]$$

22. The various forces acting on the ladders are shown in the figure.

Since the system is in equilibrium, therefore

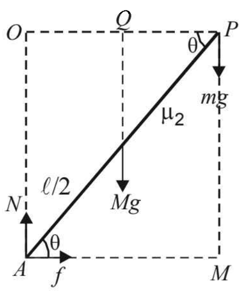
$\Sigma F_y = 0$

$\Rightarrow Mg + mg + Mg = N + N$



$$\Rightarrow N = \frac{(2M + m)g}{2} \quad \text{... (i)}$$

Considering the rotational equilibrium of one ladder as shown in figure. Calculating torques about P



$Mg \times PQ + f \times PM = N \times QP$

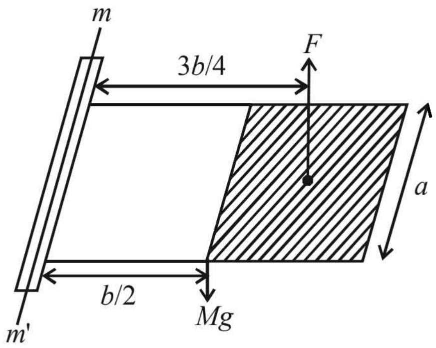
$$\Rightarrow Mg \times \frac{L}{2}\cos\theta + f \times L\sin\theta = NL\cos\theta$$

$$\Rightarrow f = \frac{NL\cos\theta - \frac{MgL}{2}\cos\theta}{L\sin\theta} = N\cot\theta - \frac{Mg}{2}\cot\theta$$

$$\Rightarrow f = \left[\left(\frac{2N + m}{2}\right)g - \frac{Mg}{2}\right]\cot\theta$$

$$\Rightarrow f = \left[(M + m)\frac{g}{2}\right]\cot\theta$$

23. **KEY CONCEPT** Since the plate is held horizontal therefore net torque acting on the plate is zero.



$$\Rightarrow Mg \times \frac{b}{2} = F \times \frac{3b}{4} \quad \text{... (i)}$$

$$F = n\frac{dp}{dt}(\text{Area}) = n \times (2mv) \times a \times \frac{b}{2} \quad \text{... (ii)}$$

From (i) and (ii)

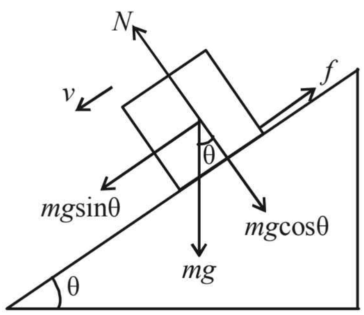
$$Mg \times \frac{b}{2} = n \times (2mv) \times a \times \frac{b}{2} \times \frac{3b}{4}$$

$$\Rightarrow 3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \frac{3 \times 2}{4}$$

$$\Rightarrow v = 10 \text{ m/s}$$

F. Match the Following

1. **A → (p,t) ; B → (q,s,t) ; C → (p,r,t) D → (q, p)**



As the velocity is constant

$f = mg\sin\theta$ ..... (i)

But $f = \mu N = \mu mg\cos\theta$ ..... (ii)

From (i) and (ii)

$\mu mg\cos\theta = mg\sin\theta \Rightarrow \mu = \tan\theta$

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The force by X on Y is the resultant of f and N and is equal to

$$\sqrt{f^2+N^2}=\sqrt{\mu^2N^2+N^2}=\sqrt{\mu^2+1}N$$

$$=(\sqrt{\tan^2\theta+1})\,mg\cos\theta=\sec\theta mg\cos\theta=mg$$

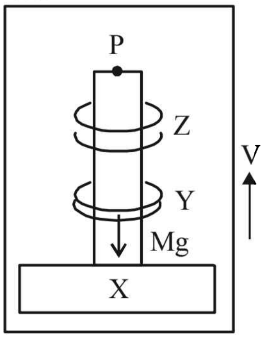
= weight of Y.

Therefore statement (a) is correct.

Now, due to the presence of frictional force between Y and X, the mechanical energy of the system $(X+Y)$ decreases continously as Y slides down.

Therefore statement (c) is correct.

(q) As the lift moves up, X also moves up and therefore the gravitational energy of X is continously increasing.



option (b) is correct.

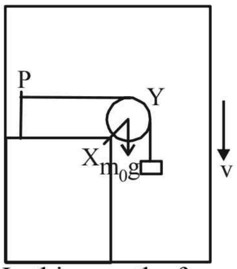
The torque of the weight of Y about P is zero as the perpendicular distance of the line of action of force from the point P is zero.

Option (d) is correct.

The force exerted by X on Y will be equal to $Mg+Mg=2mg$ where Mg is wt. of Y and Mg is the force on Y due to Z.

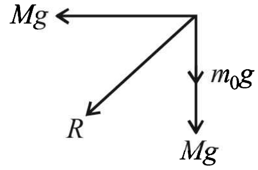
Option (a) is incorrect.

(r)



In this case the force exerted by X on Y is same as the force exerted by Y on X. The force on X due to Y is

$$R=\sqrt{(Mg)^2+(m_0+M)g]^2}\neq Mg$$



Therefore, option (a) is incorrect.

The mechanical energy of the system (X + Y) is continously decreasing as the system is coming down and its potential energy is decreasing, the kinetic energy remaining the same.

Therefore, option (c) is correct and (b) is incorrect.

The torque of the weight of Y about P is not zero.

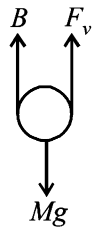
(s) The force on Y by X is equal to the wt. of liquid displaced which cannot be equal to Mg as the density of Y is greater than density of X (As Y is sinking)

Therefore, option (a) is in correct.

The gravitational potential energy of X increases continously because as Y moves down, the centre of mass of X moves up.

Therfore option (b) is correct.

(t) Sphere Y is moving with terminal velocity. Therefore, the net force on Y is zero i.e.



$$Mg=B+F_v$$

where B = buoyant force and F_v = viscous force.

$B+F_v$ are exerted by X on Y.

Therefore, option (a) is correct.

The gravitational potential energy of X is continously increasing because as Y moves down, the centre of mass of X moves up.

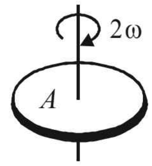
Option [b] is correct.

The mechanical energy of the system (X + Y) is continously decreasing to overcome the viscous forces.

Option (c) is correct.

G. Comprehension Based Questions

1. **(c)** For disc A



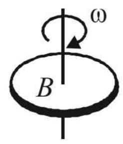
$$\frac{1}{2}kx_1^2=\frac{1}{2}I(2\omega)^2$$

$$\Rightarrow\quad kx_1^2=2I\omega^2 \qquad ...(i)$$

For disc B

$$\frac{1}{2}kx_2^2=\frac{1}{2}\times 2I\,\omega^2$$

$$\Rightarrow\quad kx_2^2=I\,\omega^2 \qquad ...(ii)$$



On dividing (i) and (ii), we get

$$\frac{k\,x_1^2}{kx_2^2}=\frac{2I\omega^2}{I\omega^2}\Rightarrow\frac{x_1}{x_2}=\sqrt{2}$$

2. **(a)** **When disc B is brought in contact with disc A**

Let ω' be the final angular velocity of both the disc rotating together. Apply conservation of angular momentum for the two disc system.

$$I(2\omega)+2I(\omega)=(I+2I)\,\omega' \Rightarrow \omega'=\frac{4}{3}\omega$$

Torque on disc A

$$\tau_A=\frac{\Delta L_A}{t}=\frac{L_f-L_i}{t}=\frac{I\times\frac{4}{3}\omega-I\times 2\omega}{t}=\frac{-2I\omega}{3t}$$

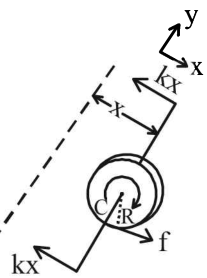
Note : The negative sign represents that the torque creates angular retardation.

3. **(b)** Loss in kinetic energy $=(K.E.)_{initial}-(K.E.)_{final}$

$$=\left[\frac{1}{2}I(2\omega)^2+\frac{1}{2}(2I)\omega^2\right]-\left[\frac{1}{2}(I+2I)\left(\frac{4}{3}\omega\right)^2\right]$$

$$=3I\omega^2-\frac{8}{3}I\omega^2=\frac{I\omega^2}{3}$$

4. **(d)** When the disc is at a distance x from the mean position (equilibrium position), the forces acting on the disc are shown in the figure



$\therefore -2kx+f=-Ma_c$ ...(1)

where a_c = acceleration of center

of mass. Also the torque acting on the disc about its center of mass C is $\tau = f \times R = I \times \alpha_c$

$$\therefore f = \frac{I\alpha}{R} = \frac{\frac{1}{2}MR^2}{R} \times \frac{a_c}{R}$$

$[\because I = \frac{1}{2}MR^2$ and $a_c = R\alpha_c$ for rolling without slipping]

$$\therefore f = \frac{1}{2}Ma_c \qquad \text{...(ii)}$$

From (i) & (ii)

$$-2kx + \frac{1}{2}Ma_c = -Ma_c$$

$$\Rightarrow \frac{3}{2}Ma_c = 2kx \quad \Rightarrow Ma_c = \frac{4kx}{3}$$

$\Rightarrow$ Net external force acting on the disc when its centre of mass is at displacement x with respect to the equilibrium position $= \frac{4kx}{3}$ directed towards the equilibrium.

5. **(d)** As derived in ans 4.

$$|F_{net}| = \frac{4k}{3}x$$

For S.H.M. $\quad |F_{net}| = M\omega^2 x$

$$\therefore M\omega^2 = \frac{4k}{3} \Rightarrow \omega = \sqrt{\frac{4k}{3M}} \qquad \text{...(iii)}$$

6. **(c)** From (i) & (ii)

$$\Rightarrow -2kx + f = -2f \Rightarrow \quad f = \frac{2k}{3} \times x$$

We see that the frictional force depends on x. As *x* increases, f increases. Also, the frictional force is maximum at x = A where A is the amplitude of S.H.M. Therefore the maximum frictional force

$$f_{max} = \frac{2k}{3} \times A$$

The force should be utmost equal to the limiting friction (μMg) for rolling without slipping.

$$\therefore \mu Mg = \frac{2k}{3} \times A \qquad \text{....(iv)}$$

For S.H.M. Velocity amplitude = $A\omega$ $\quad \therefore V_o = A\omega$

$$\therefore V_o = \frac{3\mu Mg}{2k}\omega \qquad \text{from (iv)}$$

$$\therefore V_o = \frac{3\mu Mg}{2k} \times \sqrt{\frac{4k}{3M}} \qquad \text{from (iii)}$$

$$\therefore V_o = \mu g\sqrt{\frac{3M}{k}}$$

7. **(a)** Axis of rotation is parallel to z-axis.

8. **(d)** Since the body is rigid, ω is same for any point of the body.

H. Assertion & Reason Type Questions

1. **(d)** **Statement 1 :** For velocity of centre of mass to remain constant the net force acting on a body must be zero. Therefore the statement 1 is false.

Statement 2 : The linear momentum of an isolated system remains constant. This statement is true.

2. **(d)** The acceleration of a body rolling down an inclined plane is given by

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

For hollow cylinder $\frac{I}{MR^2} = \frac{MR^2}{MR^2} = 1$

For solid cylinder $\frac{I}{MR^2} = \frac{\frac{1}{2}MR^2}{MR^2} = \frac{1}{2}$

$\Rightarrow$ Acceleration of solid cylinder is more than hollow cylinder and therefore solid cylinder will reach the bottom of the inclined plane first.

$\therefore$ Statement -1 is false

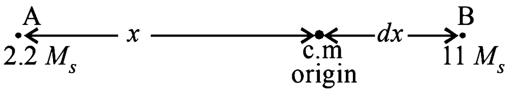
• Statement - 2

In the case of rolling there will be no heat losses. Therefore total mechanical energy remains conserved. The potential energy therefore gets converted into kinetic energy. In both the cases since the initial potential energy is same, the final kinetic energy will also be same. Therefore statement -2 is correct.

I. Integer Value Correct Type

1. **6**

Let the center of mass of the binary star system be at the origin. Then



$$0 = \frac{2.2M_s(-x) + 11M_s(d-x)}{2.2M_s + 11M_s}$$

$$\Rightarrow 0 = 2.2\,M_s(-x) + 11M_s(d-x) \Rightarrow x = \frac{5d}{6}$$

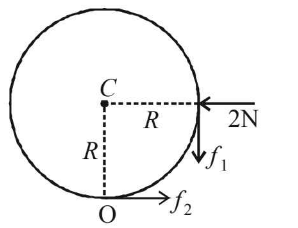
For a binary star system, angular speed ω about the centre of mass is same for both the stars.

$$\therefore \frac{L_{Total}}{L_B} = \frac{2.2M_s\left(\frac{5d}{6}\right)^2\omega + 11M_s\left(\frac{d}{6}\right)^2 \times \omega}{11M_s\left(\frac{d}{6}\right)^2 \times \omega} = \mathbf{6}$$

2. **4**

Under the influence of the force of stick (2N), the point of contact O of the ring with ground tends to slide. But the frictional force f_2 does not allow this and creates a torque which starts rolling the ring. A friction force f_1 also acts between the ring & the stick.

Applying $F_{net} = ma$ in the horizontal direction. We get



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$2 - f_2 = 2 \times 0.3$ $\therefore f_2 = 1.4\,N$

Applying $\tau = I\alpha$ about C we get

$$(f_2 - f_1) R = I\alpha = I\frac{a}{R} \quad [\because \text{For rolling } a = R\alpha]$$

$$\therefore [1.4 - \mu \times 2] \times 0.5 = 2 \times (0.5)^2 \times \frac{0.3}{0.5} \quad [\because I = MR^2]$$

$\therefore \mu = 0.4$

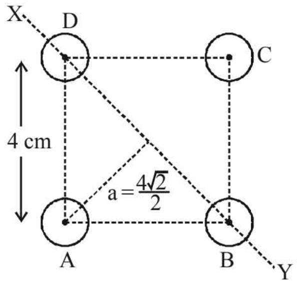
Given $\mu = \frac{P}{10}$ $\therefore P = 4$

3. **9**

Let the four spheres be A, B, C, & D

$$I_{XY} = I_1 + I_B + I_C + I_D = 2I_A + 2I_B$$

$$= 2\left[\frac{2}{5}MR^2 + Ma^2\right] + 2\left[\frac{2}{5}MR^2\right]$$



$$= 4 \times \frac{2}{5}MR^2 + 2Ma^2 = M\left[\frac{8}{5}R^2 + 2(a)^2\right]$$

$$= 0.5\left[\frac{8}{5} \times \left(\frac{\sqrt{5}}{2}\right)^2 + 2 \times 8\right] \times 10^{-4}$$

$$= 0.5[2 + 16] \times 10^{-4} = 9 \times 10^{-4}$$

$\therefore$ N = 9

4. **3** Let σ be the surface mass density. Then

$$I_O = \frac{1}{2}\sigma[\pi(2R)^2] \times (2R)^2 - \left[\frac{1}{2}(\sigma\pi R^2)^2 + \sigma(\pi R^2) \times R^2\right]$$

$$= \frac{13}{2}\pi\sigma R^4$$

$$I_P = 8\pi\sigma R^4 + \sigma\pi(2R)^2 \times (2R)^2 - \left[\frac{1}{2}\sigma(\pi R^2)R^2 + \sigma(\pi R^2)\left(\sqrt{(2R)^2 + R^2}\right)^2\right]$$

$$= 24\pi\sigma R^4 - 5.5\sigma\pi R^4 = 18.5\pi\sigma R^4$$

$$\therefore \frac{I_P}{I_O} = \frac{18.5\pi\sigma R^4}{\frac{13}{2}\pi\sigma R^4} = \frac{37}{13} \approx 3$$

5. **8** Applying conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$

$$\therefore \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\frac{1}{2}MR^2 \times \omega_1}{\left\{\frac{1}{2}MR^2 + 2[2mr^2]\right\}}$$

$$= \frac{\frac{1}{2} \times 50 \times 0.4 \times 0.4 \times 10}{\frac{1}{2} \times 50 \times 0.4 \times 0.4 + 2[2 \times 6.25 \times 0.2 \times 0.2]} = \frac{40}{4+1} = 8\text{ rad/s}$$

6. (4) By conservation of angular momentum

$2(mvr) = I\omega$

$$2 \times 0.05 \times 9 \times 0.25 = \frac{1}{2} \times 0.45 \times (0.5)^2 \times \omega$$

$\therefore \omega = 4\text{ rad s}^{-1}$

7. **(2)** $3\left[F \times r \times \frac{1}{2}\right] = I\alpha$

$$3 \times 0.5 \times 0.5 \times \frac{1}{2} = \frac{1}{2} \times 1.5 \times 0.5 \times 0.5 \times \alpha$$

$\Rightarrow \alpha = 2\text{ rad s}^{-1}$

$\omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + 2 \times 1 = 2\text{ rad s}^{-1}$

8. **(7)** Total kinetic energy of a rolling disc $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$K.E = \frac{3}{4}mv^2$$

For surface *AB*

$k.E_i$ + loss in gravitational potential energy $= K.E_f$

$$\frac{3}{4}m(3)^2 + mg(30) = \frac{3}{4}mV_B^2 \quad \text{...(i)}$$

For surface *CD*

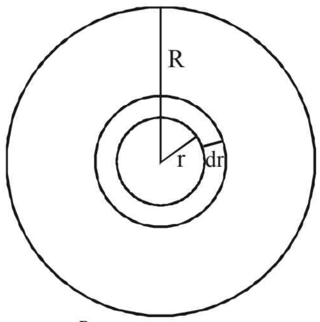
$$\frac{3}{4}m(v_2)^2 + mg(27) = \frac{3}{4}mV_D^2 \quad \text{...(ii)}$$

Given $V_B = V_D$. Therefore from (i) and (ii)

$$\frac{3}{4}m(3)^2 + mg \times 30 = \frac{3}{4}m(v_2)^2 + mg \times 27$$

$\therefore V_2 = 7$

9. **(6)** $I = \int_0^R (dm)r^2$



$$\therefore I = \int_0^R \rho \times 4\pi r^2\, dr \times r^2$$

$$\therefore I = 4\pi\int_0^R \rho\, r^4\, dr$$

$$\therefore I_A = 4\pi\int_0^R k\frac{r}{R} \times r^4\, dr = \frac{4\pi K}{R}\int_0^R r^5\, dr$$

$$= \frac{4\pi K}{R}\left(\frac{R^6}{6}\right) = 4\pi K\frac{R^5}{6}$$

$$I_B = 4\pi\int_0^R K\left(\frac{r}{R}\right)^5 r^4\, dr = \frac{4\pi K}{R^5} \times \frac{R^{10}}{10} = 4\pi K\frac{R^5}{10}$$

$$\therefore \frac{I_B}{I_A} = \frac{6}{10} \Rightarrow n = 6$$

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Section-B JEE Main/ AIEEE

1. (c) When two small spheres of mass m are attached gently, the external torque, about the axis of rotation, is zero and therefore the angular momentum about the axis of rotation is constant.

$$\therefore I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$$

Here $I_1 = \frac{1}{2}MR^2$ and $I_2 = \frac{1}{2}MR^2 + 2mR^2$

$$\therefore \omega_2 = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2mR^2}\times\omega_1 = \frac{M}{M+4m}\omega_1$$

2. (b) For negotiating a circular curve on a levelled road, the maximum velocity of the car is $v_{max} = \sqrt{\mu rg}$

Here $\mu = 0.6, r = 150$ m, $g = 9.8$

$\therefore v_{max} = \sqrt{0.6\times150\times9.8} \simeq 30\text{m/s}$

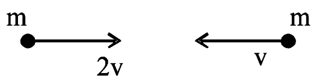
3. (b) The velocity of efflux is given

$v = \sqrt{2gh}$

Where h is the height of the free surface of liquid from the hole

$\therefore v = \sqrt{2\times10\times20} = 20\text{m/s}$

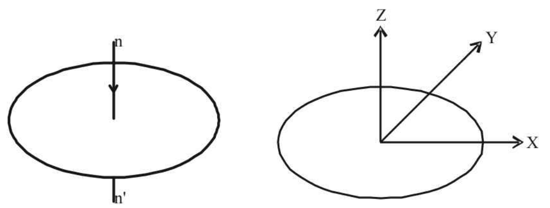
4. (c) The velocity of centre of mass of two particle system is given by



$$v_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m(2v) + m(-v)}{m+m} = \frac{v}{2}$$

5. (d) This is a case of sliding (the plane being frictionless) and therefore the acceleration of all the bodies is same ($g \sin\theta$).

6. (a) M. I of a circular wire about an axis nn' passing through the centre of the circle and perpendicular to the plane of the circle $= MR^2$



As shown in the figure, X-axis and Y-axis lie in the plane of the ring. Then by perpendicular axis theorem

$I_X + I_Y = I_Z$

$\Rightarrow 2I_X = MR^2$ $[\because I_X = I_Y$ (by symmetry) and $I_Z = MR^2]$

$\therefore I_X = \frac{1}{2}MR^2$

7. (d) Angular momentum (L)

= (linear momentum) × (perpendicular distance of the line of action of momentum from the axis of rotation)

$= mv\times r$ [Here $r = 0$ because the line of action of momentum passes through the axis of rotation]

$= mv \times 0$

$= 0$

8. (d) We know that density $(d) = \frac{mass(M)}{volume(V)}$

$\therefore M = d\times V = d\times(\pi R^2\times t)$.

The moment of inertia of a disc is given by $I = \frac{1}{2}MR^2$

$\therefore I = \frac{1}{2}(d\times\pi R^2\times t)R^2 = \frac{\pi d}{2}t\times R^4$

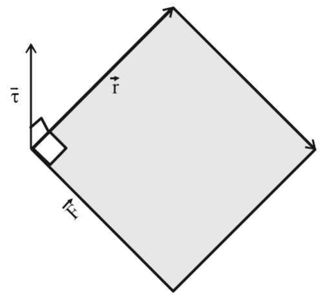
$$\therefore \frac{I_X}{I_Y} = \frac{t_X R_X^4}{t_Y R_Y^4} = \frac{t\times R^4}{\frac{t}{4}\times(4R)^4} = \frac{1}{64}$$

9. (a) $K.E. \frac{1}{2}I\omega^2$, but $L = I\omega \Rightarrow I = \frac{L}{\omega}$

$\therefore K.E. = \frac{1}{2}\frac{L}{\omega}\times\omega^2 = \frac{1}{2}L\omega$

$$\therefore \frac{K.E}{K.E'} = \frac{L\times\omega}{L'\times\omega'} \Rightarrow \frac{K.E}{\frac{K.E}{2}} = \frac{L\times\omega}{L'\times2\omega} \quad \therefore L' = \frac{L}{4}$$

10. (d) We know that $\vec{\tau} = \vec{r}\times\vec{F}$



The angle between $\vec{\tau}$ and $\vec{r}$ is 90° and the angle between $\vec{\tau}$ and $\vec{F}$ is also 90°. We also know that the dot product of two vectors which have an angle of 90° between them is zero. Therefore (d) is the correct option.

11. (b) Angular momentum will remain the same since external torque is zero.

12. (a) The moment of inertia of solid sphere A about its diameter

$I_A = \frac{2}{5}MR^2$.

The moment of inertia of a hollow sphere B about its diameter $I_B = \frac{2}{3}MR^2$. $\quad \therefore I_A < I_B$

13. (a) Does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

14. (c) The disc may be assumed as combination of two semi circular parts.

Let I be the moment of inertia of the uniform semicircular disc

$$\Rightarrow 2I = \frac{2Mr^2}{2} \Rightarrow I = \frac{Mr^2}{2}$$

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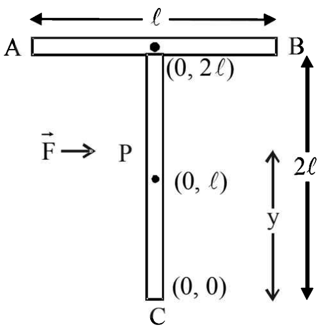
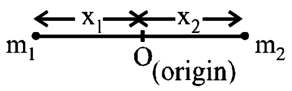
15. (d)



To have linear motion, the force $\vec{F}$ has to be applied at centre of mass.
i.e. the point '*P*' has to be at the centre of mass

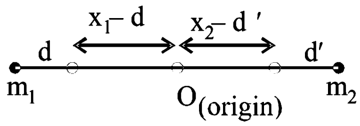
$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m \times 2\ell + 2m \times \ell}{3m} = \frac{4\ell}{3}$$

16. (c) Initially,



$$0 = \frac{m_1(-x_1) + m_2x_2}{m_1 + m_2} \Rightarrow m_1x_1 = m_2x_2 \quad ...(1)$$

Finally,
The centre of mass is at the origin

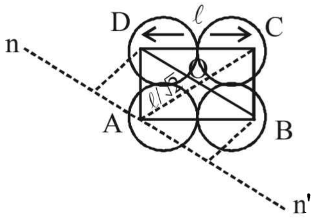


$$\therefore 0 = \frac{m_1(d - x_1) + m_2(x_2 - d')}{m_1 + m_2}$$

$$\Rightarrow 0 = m_1d - m_1x_1 + m_2x_2 - m_2d' \Rightarrow d' = \frac{m_1}{m_2}d$$

[From (1).]

17. (c)



$I_{nn'}$ = *M.I* due to the point mass at *B* +
M.I due to the point mass at *D* +
M.I due to the point mass at *C*.

$$I_{nn'} = 2 \times m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$$

$$= m\ell^2 + 2m\ell^2 = 3m\ell^2$$

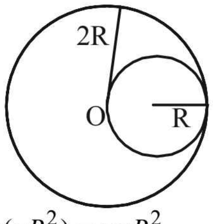
18. (b) Torque $\vec{\tau} = \vec{r} \times \vec{F} = (\hat{j} - \hat{i}) \times (-F\hat{k}) = -F(\hat{i} + \hat{j})$

19. (d) Applying conservation of angular momentum $I'\omega' = I\omega$

$$(mR^2 + 2MR^2)\omega' = mR^2\omega$$

$$\Rightarrow \omega' = \omega\left[\frac{m}{m + 2M}\right]$$

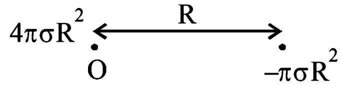
20. (b) Let the mass per unit area be σ.
Then the mass of the complete disc



$= \sigma[\pi(2R)^2] = 4\pi\sigma R^2$

The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$

Let us consider the above situation to be a complete disc of radius 2*R* on which a disc of radius *R* of negative mass is superimposed. Let *O* be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :



$$x_{c.m} = \frac{(4\pi\sigma R^2) \times 0 + (-\pi\sigma R^2)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore \; x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2} \qquad \therefore \; x_{c.m} = -\frac{R}{3} \Rightarrow \alpha = \frac{1}{3}$$

21. (b) This is a standard formula and should be memorized.

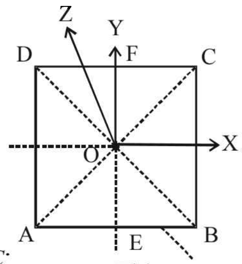
$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

22. (d) We know that $\vec{\tau_c} = \frac{d\vec{L_c}}{dt}$

where $\vec{\tau_c}$ torque about the center of mass of the body and $\vec{L_c}$ = Angular momentum about the center of mass of the body. Central forces act along the center of mass. Therefore torque about center of mass is zero.

When $\vec{\tau_c} = 0$ then $\vec{L_c}$ = constt.

23. (d) By the theorem of perpendicular axes,
$I_z = I_x + I_y$ or, $I_z = 2I_y$
($\because I_x = I_y$ by symmetry of the figure)



$$\therefore \; I_{EF} = \frac{I_z}{2} \quad ...(i)$$

Again, by the same theorem
$I_z = I_{AC} + I_{BD} = 2I_{AC}$
($\therefore I_{AC} = I_{BD}$ by symmetry of the figure)

$$\therefore \; I_{AC} = \frac{I_z}{2} \quad ...(ii)$$

From (i) and (ii), we get $I_{EF} = I_{AC}$.

24. (a)
- When $n = 0$, $x = k$ where *k* is a constant. This means that the linear mass density is constant. In this case the centre of mass will be at the midelle of the rod ie at *L*/2. Therefore (c) is ruled out
- *n* is positive and as its value increases, the rate of increase of linear mass density with increase in *x* increases. This shows that the centre of mass will shift towards that end of the rod where $n = L$ as the value of *n* increases. Therefore graph (b) is ruled out.
- The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$ *Here* $\frac{x}{L} \le 1$

With increase in the value of *n*, the centre of mass shift

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towards the end $x = L$ such that first the shifting is at a higher rate with increase in the value of n and then the rate decreases with the value of n.
These characteristics are represented by graph (a).

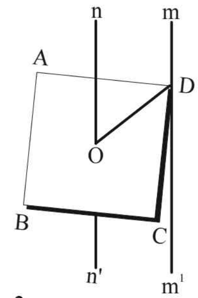
$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\lambda\,dx)}{\int_0^L \lambda\,dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n .xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{k\,x^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

For $n = 0, x_{CM} = \frac{L}{2}$; $n = 1$,

$x_{CM} = \frac{2L}{3}$; $n = 2$, $x_{CM} = \frac{3L}{4}$;....

25. **(d)** $I_{nn'} = \frac{1}{12}m(a^2 + a^2) = \frac{ma^2}{6}$



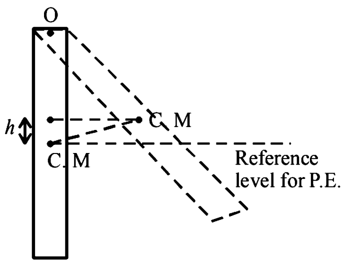
Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

According to parallel axis theorem

$$I_{mm'} = I_{nn'} + m\left(\frac{a}{\sqrt{2}}\right)^2$$

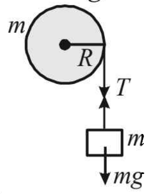
$$= \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{ma^2 + 3ma^2}{6} = \frac{2}{3}ma^2$$

26. **(c)** The moment of inertia of the rod about O is $\frac{1}{3}m\ell^2$. The maximum angular speed of the rod is when the rod is instantaneously vertical. The energy of the rod in this condition is $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in its extreme portion, its angular velocity is zero momentarily. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises



$$\therefore\ mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2 \Rightarrow h = \frac{\ell^2\omega^2}{6g}$$

27. **(b)** For translational motion,
$mg - T = ma$ .....(1)
For rotational motion,



$T.R = I\alpha = I\frac{a}{R}$ ....(2)

Solving (1) & (2),

$$a = \frac{mg}{\left(m + \frac{I}{R^2}\right)} = \frac{mg}{m + \frac{mR^2}{2R^2}} = \frac{2mg}{3m} = \frac{2g}{3}$$

28. **(c)** As insect moves along a diameter, the effective mass and hence the M.I. first decreases then increases so from principle of conservation of angular momentum, angular speed, first increases then decreases.

29. **(a)** $F = 20t - 5t^2$

$$\therefore\ \alpha = \frac{FR}{I} = 4t - t^2 \Rightarrow \frac{d\omega}{dt} = 4t - t^2$$

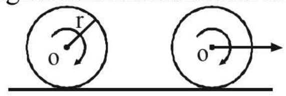
$$\Rightarrow \int_0^\omega d\omega = \int_0^t \left(4t - t^2\right)dt$$

$$\Rightarrow \omega = 2t^2 - \frac{t^3}{3} \text{ (as } \omega = 0 \text{ at } t = 0, 6s)$$

$$\int_0^\theta d\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right)dt$$

$$\Rightarrow \theta = 36 \text{ rad} \Rightarrow n = \frac{36}{2\pi} < 6$$

30. **(c)** From conservation of angular momentum about any fix point on the surface,

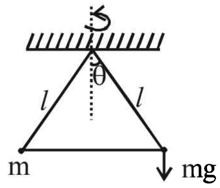


$mr^2\omega_0 = 2mr^2\omega$

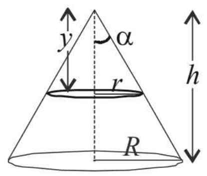
$\Rightarrow \omega = \omega_0/2 \Rightarrow v = \frac{\omega_0 r}{2}$ $[\because v = r\omega]$

31. **(c)** Torque working on the bob of mass m is, $\tau = mg \times \ell \sin\theta$. (Direction parallel to plane of rotation of particle)

As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

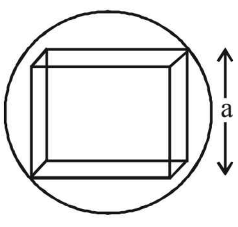


32. **(d)** $y_{cm} = \frac{\int y\,dm}{\int dm}$



$$= \frac{\int_0^h \pi r^2 dy\rho \times y}{\frac{1}{3}\pi R^2 h\rho} = \frac{3h}{4}$$

33. **(a)** Here $a = \frac{2}{\sqrt{3}}R$



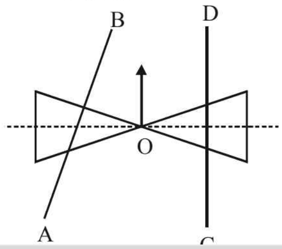
Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi. \quad M' = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about the given axis,

$$I = \frac{M'a^2}{6} = \frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

34. **(c)** As shown in the diagram, the normal reaction of AB on roller will shift towards O. This will lead to tending of the system of cones to turn left.



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